[*The following simplifies the material that starts on the middle of page 206, and runs to line 3 on page 207.*]

Why does this rule work? The key is that the equation $A\mathbf{x} = \mathbf{0}$ has exactly the same solutions as $U\mathbf{x} = \mathbf{0}$; the solution space is not affected by elementary row operations. Thus A and U have the same nullspace.

A statement of linear dependency

$$\text{column four} = (2 \times \text{column one}) + \text{column three}$$

or equivalently

$$-(2 \times \text{column one}) - \text{column three} + \text{column four} = \mathbf{0}$$

is the same as saying that the vector

$$\begin{bmatrix} -2 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

is in the nullspace. So it is true for A if and only if it is true for U.

[*Also the following, from page 210, could reasonably go here.*]

Theorem 4 For any matrix,

$$\text{dimension of nullspace} + \text{dimension of column space} = \text{the number of columns.}$$

Proof Let r be the number of pivot entries when we put the matrix in reduced row-echelon form. Then

$$\text{dimension of nullspace} = \text{the number of columns} - r$$

and

$$\text{dimension of column space} = r.$$

Add. ⊣

This theorem will reappear in §5.3 (page 255), in more abstract form.