METHODS FOR CHAPTER 6

• To calculate the (usual) inner product in $\mathbb{R}^n$: $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{y}^{\mathrm{T}}\mathbf{x}$.
  To calculate the (usual) inner product in $\mathbb{C}^n$: $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{y}^{*}\mathbf{x}$.

• To find $(\mathrm{sp}(S))^{\perp}$: Solve the system $\{\mathbf{v} \perp \mathbf{s} \mid \mathbf{s} \in S\}$ for $\mathbf{v}$. Often the theorem that

$$(\text{column space of } A)^{\perp} = \text{nullspace of } A^{*}$$

is useful.

• To find the coordinate vector $[\![\mathbf{v}]\!]_{\mathcal{B}}$ relative to an orthonormal ordered basis $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$: We can proceed as before, or we can use the Fourier coefficients:

$$[\![\mathbf{v}]\!]_{\mathcal{B}} = \begin{bmatrix} \langle \mathbf{v}, \mathbf{b}_1 \rangle \\ \vdots \\ \langle \mathbf{v}, \mathbf{b}_n \rangle \end{bmatrix}$$

• To find the representing matrix $[\![T]\!]_{\mathcal{B}}^{\mathcal{B}}$ relative to an orthonormal ordered basis $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$: We can proceed as in Chapter 5, or we can use

$$([\![T]\!]_{\mathcal{B}}^{\mathcal{B}})_{ij} = \langle T(\mathbf{b}_j), \mathbf{b}_i \rangle.$$

• To find the orthogonal projection $\mathbf{p}$ of a vector $\mathbf{v}$ onto an m-dimensional subspace W having basis $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_m\}$:

$$\mathbf{p} = \sum_{j=1}^{m} \frac{\langle \mathbf{v}, \mathbf{b}_j \rangle}{||\mathbf{b}_j||^2} \mathbf{b}_j \qquad \text{for an } \textit{orthogonal} \text{ basis}$$

$$\mathbf{p} = \sum_{j=1}^{m} \langle \mathbf{v}, \mathbf{b}_j \rangle \mathbf{b}_j \qquad \text{for an } \textit{orthonormal} \text{ basis}$$

• To replace linearly independent vectors $\mathbf{w}_1, \mathbf{w}_2, \ldots$ by nonzero orthogonal vectors $\mathbf{u}_1, \mathbf{u}_2, \ldots$ with the same span:

$$\mathbf{u}_{k+1} = \mathbf{w}_{k+1} - (\text{the projection of } \mathbf{w}_{k+1} \text{ onto } \mathrm{sp}\{\mathbf{u}_1, \ldots, \mathbf{u}_k\})$$

• (§7.3) To find an orthonormal basis for diagonalizing a symmetric linear operator (or a symmetric matrix): Proceed as in §§7.1–7.2 to make a basis of eigenvectors, with the additional step of obtaining an orthonormal basis for each individual eigenspace E_{λ}.