METHODS FOR CHAPTER 4

• To test whether a set S of vectors is a subspace of a vector space:

Determine whether S is nonempty (check to see if $\mathbf{0} \in S$). Determine whether S is closed under scalar multiplication and vector addition.

• To test whether $\mathbf{u} \in \mathrm{sp}(\mathbf{v}_1, \ldots, \mathbf{v}_n)$:

Determine whether the equation $\alpha_1\mathbf{v}_1 + \cdots + \alpha_n\mathbf{v}_n = \mathbf{u}$ has any solution for the α_i's. (Such a vector equation, in an m-dimensional space, generally converts to a system of m linear equations in the n unknowns. This system can then be solved by the methods of Chapter 2.)

• To test whether $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are linearly dependent:

See whether the equation $\alpha_1\mathbf{v}_1 + \cdots + \alpha_n\mathbf{v}_n = \mathbf{0}$ has nontrivial solutions for the α_i's. In some cases, the Wronskian is useful.

• To calculate the coordinate vector $[\![\mathbf{v}]\!]_{\mathcal{B}}$:

Solve the equation $\alpha_1\mathbf{b}_1 + \cdots + \alpha_n\mathbf{b}_n = \mathbf{v}$ for the α_i's.

• To calculate the nullspace of a matrix A:

Solve the equation $A\mathbf{x} = \mathbf{0}$ for $\mathbf{x}$. The dimension of the nullspace (the *nullity* of A) is the number of columns minus the rank.

• To find a basis for the column space of A:

One method is to use the columns of A corresponding to the pivot entries in the row-equivalent matrix U in reduced row-echelon form. The dimension of the column space is the rank of the matrix.

• To find a basis for the row space of A:

One method is to use the nonzero rows of a row-equivalent matrix U in reduced row-echelon form.