What do mathematicians do?

- Applications: science, engineering, computers, statistics, . . .

- Research: new knowledge in mathematics

- Teaching: like being a student, only in reverse

Mathematics is the science of measurement. The goal is to *understand*.

Life without 0

Standard names for positive whole numbers

- Ten digits: $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$
- Place-value notation

Zeroless names for positive whole numbers

- Ten digits: $1, 2, 3, 4, 5, 6, 7, 8, 9, X$
- Place-value notation

$4070 \qquad 3X6X$

- We have a name for every positive whole number.

- No duplicate names, no "leading zeros," no "003 = 3."

old	new
317	317
20	$1X$
4070	$3X6X$
	$X5$
130	
	XX
2010	

Arithmetic

$$\begin{array}{cc} 9 & X \\ + & 2 \\ - & - \\ X & 2 \end{array}$$

$$\begin{array}{ccc} X & 9 & X \\ & + & X \\ - & - & - \\ X & X & X \end{array}$$

$$\begin{array}{ccccc} & 3 & X & 7 \\ - & 1 & X & 3 \\ - & - & - & - \\ & 1 & X & 4 \end{array}$$

$$\begin{array}{cccc} & X & 3 & 1 \\ & & \times & X \\ - & - & - & - \\ X & 2 & X & X \end{array}$$

The point: We can understand 0 better by thinking about life *without* it.

- Positive whole numbers: Ok, we could live without 0. We could handle fractions and even negative numbers.

- Zero: Oops, no name for zero.

- Decimals: Disaster—zeroless notation does not allow decimals.

Conclusion: 0 is our friend, and now we know how we would miss it.