1. Show that every non-empty $\Sigma^1_1$ set contains an $x$ with $\omega_1^{ck(x)} = \omega_1^{ck}$. (Recall: $\omega_1^{ck(x)}$ is the sup of the ordinals “recursive in $x$” – i.e. isomorphic to some well ordering recursive in $x$).

2. Show that if $\delta < \omega_1^{ck}$ is an infinite ordinal, then there is a recursive well ordering isomorphic to $\delta$. (Warning! This is NOT a tautology. We defined $\omega_1^{ck}$ to be the supremum of such ordinals. This exercise is asking you to show that the class of order types realisable as a recursive well ordering of $\omega$ has no gaps.)

3. Let $X_{\text{low}}$ be the set of $x \in 2^\omega$ which are low ($\omega_1^{ck(x)} = \omega_1^{ck}$). Equip this with the topology generated by the $\Sigma^1_1$ sets.

Show this space is Polish.