Math 223S: Topics in Set Theory

Time: Mon, Wed, Fri, 1pm-2pm

Place: MSB 5137

Instructor: Greg Hjorth, MSB 7340

Office hours: Tentatively planned for Monday 2pm-3pm, Wednesday 10:30am-12:30pm. In any case, I should be freely available straight after lectures either for shorter questions or to schedule an appointment. You can also grab me after the Logic Seminar on Friday afternoons.

Assessment: For students who want to get an A, there will be three or four homework assignments throughout the quarter.

Reference: There are no required texts for this course, and the lectures should be completely adequate in their own right. However if you want secondary references, then I suggest Jech's *Set Theory*.

Content: We will develop the basic ideas surrounding measurable cardinals and sharps, and illustrate some of the concepts which lie at the heart of the modern theory of large cardinals and fine structure.

The very first goal will be a careful and thorough proof of Jensen's *covering lemma*. This pivotal theorem states, in some sense, a kind of dichotomy theorem: either Gödel's L approximates V or is trivially small. It is the basis for many consistency results, such as PFA implying the existence of inner models with measurable cardinals or the failure of the singular cardinal hypothesis having large cardinal strength. In order to reach the covering lemma, we will need to develop the theory of sharps. I plan to motivate that theory by discussing earlier concepts in the history of set theory, such as clubs, stationary sets, measurable cardinals, and Scott's theorem on elementary embeddings.

After the covering lemma, the choice will be dictated by the interest and background of the class. One possible subject would be the Martin-Solovay tree. Another would be iteration trees, which, if a sufficient number of people are familiar with forcing could be lead in to a discussion of Woodin's extender algebra and its application to descriptive set theory.

Prerequisites: A thorough familiarity with the constructible universe, L, as found in 220abc.