Homework V

1. Use Stokes Theorem to show that if M is a compact (connected) orientable manifold, then $H^n_{deR}(M, \mathbb{R}) \neq 0$ (n = dimension M).
[Suggestion: Show that, an orientation having been chosen, $\omega \to \int_M \omega$ induces a map $H^n_{deR}(M, \mathbb{R}) \to \mathbb{R}$].

2. Deduce Green's Theorem in $\mathbb{R}^2$ (for bounded open sets with smooth boundary) from Stokes' Th. for manifolds with boundary.

3. Same as prob 2 for Stokes' Theorem for bounded surfaces in $\mathbb{R}^3$ with boundary (from the general Stokes Theorem)

4. Same for the Divergence Theorem for bounded open sets in $\mathbb{R}^3$ with smooth boundary.

5. Suppose $f_0 = id, f_1 \ldots, f_N$ is a finite group of diffeomorphisms of a manifold M.
Suppose $\omega = d\theta$ for some θ on M {ω C^∞_k-form; θ $k-1$ form} (k may not be n!)
and $f_j^* \omega = \omega$ for all $j = 0, \ldots, N$.
Prove that $\exists\ \hat{\theta}$ with $f_j^* \hat{\theta} = \hat{\theta}$ all $j = 0..N$ and $d\hat{\theta} = \omega$. [You may assume that d commutes with pullbacks: $F^* d = dF^*$ for any C^∞ map F. We'll prove this soon].