Fundamental ideas for 2-forms:

Given two 1-forms (elements of $(T_pM)^*$, each p) ω_1, ω_2 we define for $v, w \in T_pM$

$$(\omega_1 \wedge \omega_2)(v,w) = \omega_1(v)\omega_2(w) - \omega_1(w)\omega_2(v)$$

Clearly this is an antisymmetric bilinear function on $T_pM \times T_pM$.

Observation: If $\theta_1, \dots, \theta_n$ is a basis for $(T_pM)^*$ then $\theta_i \wedge \theta_j$ $i<j$ is a basis for the space of all antisymmetric bilinear functions on $(T_pM) \times (T_pM)$.

Proof: Given A antisymmetric bilinear function check $A = \sum A(v_i, v_j)\, \theta_i \wedge \theta_j$ where $v_1 \dots v_n \in T_pM$ is dual to $\theta_1, \dots, \theta_n$.
Also $\theta_i \wedge \theta_j$ $i<j$ are linearly independent since

$$\Big(\sum_{i<j} a_{ij}\, \theta_i \wedge \theta_j\Big)(v_k, v_l) = a_{kl}.$$

Calculation $n=2$: $d\hat{x}_1 \wedge d\hat{x}_2 = \dfrac{\partial(\hat{x}_1, \hat{x}_2)}{\partial(x_1,x_2)}\, dx_1 \wedge dx_2$

where $(\hat{x}_1, \hat{x}_2)$ and (x_1,x_2) local coordinates and $\dfrac{\partial(\hat{x}_1,\hat{x}_2)}{\partial(x_1,x_2)}$ = Jacobian det = $\det\begin{pmatrix} \frac{\partial \hat{x}_1}{\partial x_1} & \frac{\partial \hat{x}_1}{\partial x_2} \\ \frac{\partial \hat{x}_2}{\partial x_1} & \frac{\partial \hat{x}_2}{\partial x_2} \end{pmatrix}$

If ω is a C^∞ 2-form, $\omega = f\, dx_1 \wedge dx_2 = \hat{f}\, d\hat{x}_1 \wedge d\hat{x}_2$ and if Jac det >0, then

$$\int f \text{ in } (x_1,x_2) \text{ coords} = \int \hat{f} \text{ in } (\hat{x}_1, \hat{x}_2) \text{ coords}$$