Homework I

1(a) Show that if $(P(t))' = p(t)$ then $y(t) = C e^{-P(t)}$ solves

$$y' + p(t)y = 0$$

(b) Show that if $y(t) = f(t)$ solves

$$y' + p(t)y = g(t)$$

then every solution of $y' + p(t)y = g(t)$ has the form

$$f(t) + Ce^{-P(t)}.$$

2 If the amount of radioactive material at time t is $C_1 e^{-C_2 t}$, what is the half-life of the decay?

3. If sound in an auditorium decays exponentially and takes 2 seconds to drop to one millionth of its initial level S_0, what the sound at a general time t? (In the form (initial value) $S_0 e^{-Ct}$, what is C?)

4. Suppose $p(t) \geq 0$ and $p(t) \geq 1/t$ if $t > 1$. Show that the "transient" part of the solution of $y' + p(t)y = g(t)$ (the part involving $y(0)$) really is transient, i.e.,

that that part $\to 0$ as $t \to +\infty$.

[For this problem, look at problem 1!
The $Ce^{-P(t)}$ is the transient part]

In other words, show that any two solutions
$y_1(t)$ and $y_2(t)$ of $y' + p(t)y = q(t)$
satisfy $\lim_{t \to +\infty} (y_1(t) - y_2(t)) = 0$.

5. Suppose $y(t)$ t ranging over all of $\mathbb{R}$
is not identically 0 but $y(0) = 0$.
Is it possible that $y(t)$ is a solution
of a (linear homogeneous) first order differential equation $y' + p(t)y = 0$
(on all of $\mathbb{R}$). Why or why not?
[Suggestion: Think about the uniqueness part
of the existence and uniqueness theorem].

6. Suppose $y' + p(t)y = q(t)$.
Express y'' in terms of y', y and p and q
Do the same for y'''.
Could this process be continued in principle?