Uniqueness for Given $\kappa\,(>0)$ and τ.

γ_1, γ_2 curves, arclength parameter, both define on same interval, $0 \in$ interval,
κ_1, τ_1 curvature and torsion of γ_1
κ_2, τ_2 curvature and torsion of γ_2,
T_1, N_1, B_1 frame for γ_1
T_2, N_2, B_2 frame for γ_2.

Compute: $\frac{d}{ds}\left(\langle T_1, T_2\rangle + \langle N_1, N_2\rangle + \langle B_1, B_2\rangle\right)$ by finding each $\langle\,,\rangle$ derivative separately:

$$\frac{d}{ds}\langle T_1, T_2\rangle = \left\langle \frac{dT_1}{ds}, T_2\right\rangle + \left\langle T_1, \frac{d}{ds}T_2\right\rangle$$

(1) $$= \kappa_1\langle N_1, T_2\rangle + \kappa_1\langle T_1, N_2\rangle \qquad \text{since } \kappa_2 = \kappa_1$$

$$\frac{d}{ds}\langle N_1, N_2\rangle = \left\langle \frac{dN_1}{ds}, N_2\right\rangle + \left\langle N_1, \frac{dN_2}{ds}\right\rangle$$

(2) $$= -\kappa_1\langle T_1, N_2\rangle + \tau_1\langle B_1, N_2\rangle - \kappa_1\langle N_1, T_2\rangle + \tau_1\langle N_1, B_2\rangle$$

$$\frac{d}{ds}\langle B_1, B_2\rangle = \left\langle \frac{dB_1}{ds}, B_2\right\rangle + \left\langle B_1, \frac{dB_2}{ds}\right\rangle$$

(3) $$= -\tau_1\langle N_1, B_2\rangle - \tau_1\langle B_1, N_2\rangle$$

(1) + (2) + (3) $= 0$ — all terms cancel in pairs.

So

$$\langle T_1, T_2\rangle + \langle N_1, N_2\rangle + \langle B_1, B_2\rangle \text{ is constant.}$$

Translate and rotate γ_2 so $T_1(0), N_1(0), B_1(0)$ and $T_2(0), N_2(0), B_2(0)$ are identical o.n.-frames at same point. Then $\langle T_1, T_2\rangle + \langle N_1, N_2\rangle + \langle B_1, B_2\rangle = 3$ at $s = 0$ and hence $= 3$ for all s. Cauchy-Schwarz Inequality $\Rightarrow$ $T_1 = T_2$ (and $N_1 = N_2$, $B_1 = B_2$) for all s. Hence $\gamma_1 \equiv \gamma_2$ by s-integration.