Homework VI : Due Friday, November 14

1. Problem 6 from on-line homework#5

2 Problem 7 from on-line homework#5

3. Compute the Gauss and mean curvatures of the surface $S(u,v) = (u, v, f(u,v))$ in terms of derivatives of f (do not assume $f_u = f_v = 0$).

4. Use you results from problem 3 to show that the surface $S(u,v) = (u, v, \sqrt{1-u^2-v^2})$ ($u^2 + v^2 < 1$) has Gauss curvature $= +1$ everywhere. What is its mean curvature?

5. Recall that if $S(u,v) =$ a surface obtained by rotating a curve $(f(u), g(u))$ around the x axis so that $S(u,v) = (f(u), g(u)\cos v, g(u)\sin v)$ ("surface of revolution") where $f'(u) > 0$ and $(f')^2 + (g')^2 = 1$, then the Gauss curvature $= -g''(u)/g(u)$. Use this formula to construct surfaces with Gauss curvature $\equiv +1$ ~~but~~ that are not congruent to parts of spheres. (Suggestion: We want $g''(u) = -g''(u)$. Try $g(u) = \alpha \cos u$, $\alpha > 0$ but otherwise arbitrary. What does f' need to be to get $(f')^2 + (g')^2 = 1$? Go on to find f etc.)

Go to next page

6. Recall the proof that a curve in $\mathbb{R}^2$ with κ never zero (e.g. >0) which is simple and smoothly closed bounds a convex open set in $\mathbb{R}^2$. Explore the possibility of using a similar argument to show that a closed, bounded surface in $\mathbb{R}^3$ bounds a convex open set in $\mathbb{R}^3$. (You may take for granted things you will need from topology, e.g., that a closed bounded surface $S \subset \mathbb{R}^3$ has the property that $\mathbb{R}^3 - S$ has "two pieces", i.e., is the union of two disjoint connected open sets, the "inside" and the (unbounded) "outside" (interior and exterior). You will need that a surface with Gauss curvature >0 lies locally on one side of its tangent plane, as proved in class, etc.)