Geographic profiling from kinetic models of criminal behavior

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Abstract

We consider the problem of estimating the probability density of the "anchor point" (residence, place of work, etc.) of a criminal offender given a set of observed spatial locations of crimes committed by the offender. Starting from kinetic models of criminal behavior, we derive the probability density of anchor points using the Fokker-Planck equation and Bayes' Theorem. Here geographic inhomogeneities such as housing densities and geographic barriers (bodies of water, parks, etc.) are naturally incorporated into the probability density estimate. The resulting equations are elliptic PDEs that can be solved efficiently using Multigrid or other standard computational techniques. We then test our methodology against distance to crime data provided by the Los Angeles Police Department. Our results highlight the benefits of incorporating elements of criminal behavior and geographic inhomogeneities into profiling estimates.

1 Introduction

A classical problem arising in crime science is the estimation of the probability density of the anchor point $\mathbf{z} \in \mathbb{R}^2$ of a criminal offender, given a set of observed spatial locations $\mathbf{x}_1, ..., \mathbf{x}_N \in \mathbb{R}^2$ of crimes assumed to have been committed by the individual. The anchor point could be the offender's legal residence, a friend's or family member's residence, place of work, etc. [14] and can be thought of as a staging point for criminal activity.

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In practice a score function,

$$S(\mathbf{z}) = \sum_{i=1}^{N} f(d(\mathbf{x}_i, \mathbf{z})), \tag{1}$$

is often used to determine likely locations for the anchor point [15, 4, 11, 14]. Here d is a distance metric and f is a kernel that typically decays at long distances (but may be increasing at short distances). One criticism of such an approach is that the score function in (1) is not a probability density and O'Leary argues in [14] for modeling the conditional probability density $P(\mathbf{z}|\mathbf{x}_1, ..., \mathbf{x}_N)$, instead of the score function.

Another major fault common among methods of the form (1) is that f is typically isotropic and does not take into account geographic features such as housing density or observed patterns of criminal behavior. For example, consider the residential burglary scenario depicted in Figure 1. Criminals with anchor points located at $\mathbf{z}_1$, $\mathbf{z}_2$, and $\mathbf{z}_3$ would all be assigned equal likelihood of committing the burglary at the red house in the center under a model of the form (1). However, the likelihood of the anchor point being at $\mathbf{z}_1$ should be low (in comparison to other points equidistant to the crime scene), because the offender would have to travel around the lake to get there, encountering many attractive targets along the way. Likewise, the likelihood corresponding to $\mathbf{z}_3$ should be lower than $\mathbf{z}_2$, because there are more potential targets in between that could prevent the criminal starting at $\mathbf{z}_3$ from reaching the red house without committing a burglary. Furthermore, a model of the form (1) would assign even higher likelihood to points on the lake, as every point on the lake is closer to the red house than $\mathbf{z}_1$, $\mathbf{z}_2$ or $\mathbf{z}_3$.

In this paper we take an alternative approach, deriving the probability density of the anchor point, $P(\mathbf{z}|\mathbf{x}_1, ..., \mathbf{x}_N)$, using Bayes' Theorem as suggested in [14]. This requires a model for the distribution of crime targets given an anchor point, as well as a model for the (prior) distribution of anchor points. We derive the former starting from a kinetic description of criminal behavior, along the lines of models introduced in [2, 17, 7, 21] for criminal movement and target selection. Our model takes into account geographic features not typically accounted for in standard geographic profiling models. We then indicate how such a model can be implemented in practice, including computation of the geographic profile, parameter estimation, and modeling of the prior distribution.

The outline of the paper is as follows. In Section 2, we introduce a

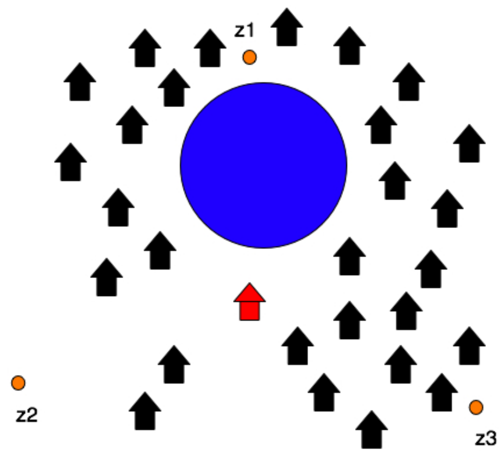

Figure 1: Residential burglary scenario with geographic inhomogeneities.

kinetic model of criminal behavior that can be used directly to compute geographic profiles or used as a starting point for more computationally efficient methods. In Section 3, we derive equations for the probability density, $P(\mathbf{z}|\mathbf{x}_1, ..., \mathbf{x}_N)$, from the kinetic models using the Fokker-Planck equation and Bayes' Theorem. The resulting equations are elliptic PDEs that allow for geographic profiles to be computed more efficiently than in the case of Monte Carlo methods applied directly to the kinetic models. In Section 4, we illustrate the effectiveness of our methodology by comparing several models fit to "distance to crime" data provided by the Los Angeles Police Department. Our findings show the importance of incorporating both geographic inhomogeneity and criminal behavior into geographic profiling estimates.

2 Kinetic model of criminal behavior

Here we develop a kinetic model of criminal behavior that will be used to derive geographic profiling estimates. The model assumes a foraging behavior for the criminal [2, 17, 9], though in general the type of model should depend on the crime type. As proposed in [17], we start by introducing a (stationary) spatial attractiveness field $A(\mathbf{y}) \geq 0$, reflecting how attractive the target positioned at $\mathbf{y}$ is to a criminal also positioned there. In the case of residential burglary, A would be proportional to housing density, $H(\mathbf{y})$, under the assumption that all houses are equally attractive. The attractiveness field will determine the rate at which criminals commit their crimes.

Letting $\mathbf{y}(t)$ denote the position of a criminal at time t, we model the movement of the criminal by the stochastic differential equation,

$$\frac{d\mathbf{y}}{dt} = \vec{\mu}(\mathbf{y}) + \sqrt{2D}\mathbf{R}_t, \tag{2}$$

where $\mathbf{R}_t$ is a white noise, i.e. $\langle \mathbf{R}_t \rangle = \mathbf{0}$ and $\langle R_t^i R_{t'}^j \rangle = \delta_{ij}\delta(t-t')$, and D is the diffusion parameter. The drift term $\vec{\mu}$ can be neglected in the case of unbiased motion or could be used to describe more complex criminal behaviors. For instance, it has been suggested that criminals may modify their movements towards regions of higher attractiveness when selecting their targets [17]. This type of behavior could be incorporated into (2) through a gradient term of the form $\vec{\mu} = \chi \nabla A$ or a non-local potential involving the attractiveness field.

Since the anchor point can be viewed as the location from which criminals begin their search for a target, we take $\mathbf{y}(0) = \mathbf{z}$ as the initial condition for (2). A crime is then committed at $\mathbf{y}(t)$ according to the killing measure $A(\mathbf{y}(t))$, the probability per unit time that the Brownian trajectory given by (2) is terminated at the space-time point $\mathbf{y}(t)$ [8, 16].

At this point geographic profiles can be computed using Monte Carlo simulation, assuming an estimate exists for the density of criminal anchor points $P(\mathbf{z})$. Given the popularity of agent based models for crime applications [17, 7, 21], such an approach may be appropriate for complex agent models that do not lend themselves to mathematical analysis. However, for models of the form (2) we show in the next section that the geographic profiles can be computed more efficiently.

3 Derivation of geographic profiling densities

Given that a criminal starts their random walk governed by (2) from the anchor point $\mathbf{z}$, the transition (survival) probability density $\rho(\mathbf{x}, t|\mathbf{z})$ [8] of the position of the criminal satisfies the Fokker-Planck equation,

$$\frac{d\rho}{dt} = \nabla \cdot (D\nabla\rho) - \nabla \cdot (\vec{\mu}(\mathbf{x})\rho) - A(\mathbf{x})\rho, \tag{3}$$

$$\rho_0 = \delta(\mathbf{x} - \mathbf{z}), \tag{4}$$

where ∇ is with respect to the variable $\mathbf{x}$ [8, 16]. Integrating (3)-(4) in time, the probability density of where the crime is committed is then determined by,

$$P(\mathbf{x}|\mathbf{z}) = A(\mathbf{x})\rho(\mathbf{x}|\mathbf{z}), \tag{5}$$

where $\rho(\mathbf{x}|\mathbf{z}) = \int_0^\infty \rho(\mathbf{x}, t|\mathbf{z})dt$ solves the elliptic partial differential equation,

$$-\nabla \cdot (D\nabla\rho) + \nabla \cdot (\vec{\mu}(\mathbf{x})\rho) + A(\mathbf{x})\rho = \delta(\mathbf{x} - \mathbf{z}). \tag{6}$$

We will refer to this equation as the "forward" equation. Given the prior distribution of criminal anchor points, $P(\mathbf{z})$, the geographic profiling distribution can then be determined using Bayes' Theorem,

$$P(\mathbf{z}|\mathbf{x}) = \frac{P(\mathbf{x}|\mathbf{z})P(\mathbf{z})}{\int_{\mathbb{R}^2} P(\mathbf{x}|\mathbf{z})P(\mathbf{z})d\mathbf{z}} = \frac{A(\mathbf{x})\rho(\mathbf{x}|\mathbf{z})P(\mathbf{z})}{\int_{\mathbb{R}^2} A(\mathbf{x})\rho(\mathbf{x}|\mathbf{z})P(\mathbf{z})d\mathbf{z}} = \frac{\rho(\mathbf{x}|\mathbf{z})P(\mathbf{z})}{\int_{\mathbb{R}^2} \rho(\mathbf{x}|\mathbf{z})P(\mathbf{z})d\mathbf{z}}. \tag{7}$$

Since $\rho(\mathbf{x}|\mathbf{z})$ is the Green's function corresponding to the linear operator on the left side of (6), for fixed $\mathbf{x}$ and varying $\mathbf{z}$ the function $f(\mathbf{z}) = \rho(\mathbf{x}|\mathbf{z})$ solves the "backward" or "adjoint" equation [1, 5, 10],

$$-\nabla \cdot (D\nabla f) - \vec{\mu}(\mathbf{z}) \cdot \nabla f + A(\mathbf{z})f = \delta(\mathbf{z} - \mathbf{x}), \tag{8}$$

where ∇ is now with respect to the variable $\mathbf{z}$. Thus the geographic profiling density can be efficiently computed in practice by solving the backward equation given by (8), where the point mass on the right hand side is located at the scene of the crime, and then multiplying by the prior distribution of anchor points and normalizing. We note that the first order derivative term changes sign going from the forward equation (6) to the backward equation (8). This has practical implications, for if criminals move up gradients of attractiveness then police investigations starting from the scene of the crime should move down gradients of attractiveness.

A similar procedure can be carried out for multiple crimes. Assuming event independence and that all crimes were committed by the same person, then the geographic profiling density for multiple events is given by,

$$P(\mathbf{z}|\mathbf{x}_1, ..., \mathbf{x}_N) = \frac{P(\mathbf{x}_1|\mathbf{z}) \cdots P(\mathbf{x}_N|\mathbf{z})P(\mathbf{z})}{\int_{\mathbb{R}^2} P(\mathbf{x}_1|\mathbf{z}) \cdots P(\mathbf{x}_N|\mathbf{z})P(\mathbf{z})d\mathbf{z}} = \frac{\Pi_{i=1}^N f_i(\mathbf{z})P(\mathbf{z})}{\int_{\mathbb{R}^2} \Pi_{i=1}^N f_i(\mathbf{z})P(\mathbf{z})d\mathbf{z}}, \tag{9}$$

where $f_i(\mathbf{z})$ solves,

$$-\nabla \cdot (D\nabla f_i) - \vec{\mu}(\mathbf{z}) \cdot \nabla f_i + A(\mathbf{z})f_i = \delta(\mathbf{z} - \mathbf{x}_i). \tag{10}$$

Also, a buffer zone could be incorporated into the forward equation through,

$$P(\mathbf{x}|\mathbf{z}) = 1_{\{|\mathbf{x}-\mathbf{z}|>\mathbf{r}\}} A(\mathbf{x}) \rho(\mathbf{x}|\mathbf{z}) \tag{11}$$

where $\rho(\mathbf{x}|\mathbf{z})$ solves the modified forward equation,

$$-\nabla \cdot (D \nabla \rho) + \nabla \cdot (\vec{\mu}(\mathbf{x}) \rho) + 1_{\{|\mathbf{x}-\mathbf{z}|>r\}} A(\mathbf{x}) \rho = \delta(\mathbf{x} - \mathbf{z}). \tag{12}$$

Here the idea is that criminals may leave a buffer zone of radius $\mathbf{r}$ around their anchor point, within which they do not commit any crimes. The backward equation is then given by,

$$-\nabla \cdot (D \nabla f) - \vec{\mu}(\mathbf{z}) \cdot \nabla f + 1_{\{|\mathbf{x}-\mathbf{z}|>r\}} A(\mathbf{z}) f = \delta(\mathbf{z} - \mathbf{x}), \tag{13}$$

and the geographic profiling density is,

$$P(\mathbf{z}|\mathbf{x}) = \frac{1_{\{|\mathbf{x}-\mathbf{z}|>r\}} f(\mathbf{z}) P(\mathbf{z})}{\int_{|\mathbf{x}-\mathbf{z}|>r} f(\mathbf{z}) P(\mathbf{z}) d\mathbf{z}}. \tag{14}$$

4 Example: burglary in Los Angeles

Next we illustrate the implementation of our methodology using burglary data collected by the Los Angeles Police Department in 2004 within a 10 km by 10 km region of the San Fernando Valley. The data consists of 45 locations, $\mathbf{x}_i$, where the burglaries occurred and the 45 corresponding residences, $\mathbf{z}_i$, of the offenders (presumed to be their anchor points).

We first fit the parameters of three competing models using maximum likelihood estimation. Here we assume that the motion of each criminal in the city is governed by the stochastic differential equation (2) with the same diffusive parameter D. In general, given historical distance to crime data on multiple offenders (each of whom committed multiple crimes), a prior distribution $\pi(D)$ for the diffusive parameter could be estimated and incorporated into the modeling framework,

$$P(\mathbf{z}|\mathbf{x}_1, ..., \mathbf{x}_N) \propto \int P(\mathbf{x}_1, ..., \mathbf{x}_N | \mathbf{z}, D) P(\mathbf{z}) \pi(D) dD, \tag{15}$$

as discussed in [14]. In each of the three models we also assume that criminals diffuse without drift ($\vec{\mu} = \mathbf{0}$).

For the first model we define the attractiveness field and the prior distribution of anchor points to be spatially homogeneous: $A(\mathbf{x}) = A_0$ and $P(\mathbf{z}) = P_0$. Such a model is similar to those used in practice, as the resulting distribution is isotropic and does not take into account housing density or other geographic inhomogeneities.

For the second model we let A be homogeneous and $P(\mathbf{z})$ be proportional to housing density, to test whether it is sufficient to use an isotropic density for $P(\mathbf{x}|\mathbf{z})$ and then to incorporate inhomogeneities through $P(\mathbf{z})$. For the third model we assume that criminals are uniformly distributed amongst all residences within the city and furthermore that all houses are equally attractive. Thus both the attractiveness field $A(\mathbf{x}) = A_0 \cdot H(\mathbf{x})$ and the prior distribution $P(\mathbf{z}) = P_0 \cdot H(\mathbf{z})$ are taken to be proportional to housing density H. In practice $P(\mathbf{z})$ could be augmented with information from police databases on the residences of past offenders, parolees, and suspects in other crimes.

Each model has one effective parameter, $\theta = D/A_0$, which is found by maximizing the log likelihood function:

$$\max_{\theta} \sum_{i=1}^{45} \log \Big(P(\mathbf{z}_i|\mathbf{x}_i) \Big). \tag{16}$$

For each event $\mathbf{x}_i$ and parameter value θ the backward equation,

$$-\nabla \cdot (D \nabla f_i) - \vec{\mu}(\mathbf{z}) \cdot \nabla f_i + A(\mathbf{z}) f_i = \delta(\mathbf{z} - \mathbf{x}_i), \tag{17}$$

is solved using Multigrid [3] on an 18 km by 18 km domain, with a 128x128 resolution and Dirichlet boundary conditions. Here a buffer of 4km on each side of the data set is used to avoid boundary effects. The log likelihood function (16) is then calculated for each parameter using,

$$P(\mathbf{z}_i|\mathbf{x}_i) = \frac{f_i(\mathbf{z}_i) P(\mathbf{z}_i)}{\int f_i(\mathbf{z}) P(\mathbf{z}) d\mathbf{z}}. \tag{18}$$

In Table 1 we list the maximized log likelihood values for each of the three models. As expected, taking A to be homogeneous severely limits the accuracy of the model since a great deal of probability density is distributed in areas where there are no houses. Interestingly, model 3 significantly outperforms model 2, even though both models incorporate geographic information and distribute probability density in realistic regions of the city. Thus it is important to accurately model criminal target selection through $P(\mathbf{x}|\mathbf{z})$, instead of using an ad hoc approach that only incorporates geographic inhomogeneities through $P(\mathbf{z})$.

Table 1: Maximized log likelihood values of competing models

model 1 $A(\mathbf{x})$ homogeneous $P(\mathbf{z})$ homogeneous	model 2 $A(\mathbf{x})$ homogeneous $P(\mathbf{z})$ inhomogeneous	model 3 $A(\mathbf{x})$ inhomogeneous $P(\mathbf{z})$ inhomogeneous
-242.726	-238.236	$\mathbf{-213.996}$

In Figure 2 we include a plot of housing density in the region of the San Fernando Valley considered in this study and in Figures 3 and 4 we plot several geographic profiles corresponding to the three different models. In Figure 2 the spatial inhomogeneity of housing density is illustrated, where there are several regions with high density and other regions, including commercial zones, parks, and mountains, void of housing. In Figure 3 we plot geographic profiles using the best fit parameters for each of the three models. Whereas model 1 fails to detect the commercial and public areas in the center of the domain, models 2 and 3 distribute probability density around these areas. The visual differences between model 2 and model 3 are more subtle, but the isotropy of $P(\mathbf{x}|\mathbf{z})$ in model 2 can be seen in the middle figure. In Figure 4, we provide examples of geographic profiles corresponding to model 3 in the case of multiple crimes and when a buffer zone is included. The probability density is much more localized in the case of multiple crimes due to the product in Equation (9). We note that this is not the case when a summation of the form (1) is used instead.

5 Discussion

We developed a new framework for geographic profiling based upon Bayes' Theorem and kinetic descriptions of criminal behavior. In the future it may prove fruitful to consider general models for the forward equation of the form,

$$-L\rho + \nabla \cdot (\vec{\mu}(\mathbf{x})\rho) + A(\mathbf{x})\rho = \delta(\mathbf{x} - \mathbf{z}). \tag{19}$$

Here L could correspond to fractional diffusion [20], as a heavy tailed distribution may be more appropriate in the case of more serious crimes (for

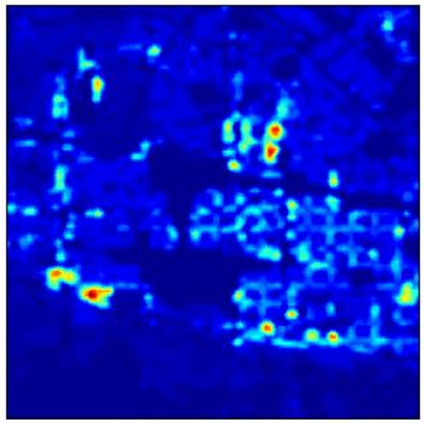

Figure 2: Housing density for the 18 km by 18 km region of the San Fernando Valley used in this study. Center regions void of housing correspond to a commercial area and a park and lower regions void of houses correspond to mountains.

which geographic profiling is typically used). Another possibility would be to model the diffusion parameter in (2) as a stochastic process, along the lines of stochastic volatility models used in financial modeling [6].

For example, in Figure 5 we plot a histogram of the distances, $|\mathbf{x}_i - \mathbf{z}_i|$, from the 2004 data set and a histogram of distances sampled from $P(\mathbf{z}|\mathbf{x}_i)$ corresponding to model 3 using the best fit parameters. Here the actual data has a heavier tail, promoting either the use of fractional diffusion or a non-atomic prior distribution, $\pi(D)$, for the diffusion parameter.

It also may be advantageous to incorporate inhomogeneous criminal mobility, i.e. $L = \nabla\cdot(D(\mathbf{x})\nabla)$, into geographic profiling estimates. For example, in the city of Los Angeles traffic could play an important role in determining the location of anchor points. Anchor points equidistant (spatially) from the crime, but with varying *time* distances from the crime, should have different probability weights. Here traffic data (for example from Google) could be used to reconstruct the spatial diffusion field $D(\mathbf{x})$. Furthermore, geographic obstacles such as parks and bodies of water could also be included in such estimates, as criminals must diffuse around these objects (not through) to reach targets on the other side.

For other types of crime where housing density does not play a role in target selection, such as person to person crimes, auto theft, etc., the attractiveness field can be estimated from historical crime data. First, the marginal probability density of crime $P_c(\mathbf{x})$, which for a given model satisfies,

$$P_c(\mathbf{x}) = \int_{\mathbb{R}^2} A(\mathbf{x})\rho(\mathbf{x}|\mathbf{z})P(\mathbf{z})d\mathbf{z}, \tag{20}$$

can be inferred from aggregate spatial crime data using density estimation techniques for spatial point processes [13, 19]. Then, for a given estimate $\hat{P}_c(\mathbf{x})$, the attractiveness field A could be estimated by minimizing,

$$\int_{\mathbb{R}^2} \left(\hat{P}_c(\mathbf{x}) - \int_{\mathbb{R}^2} A(\mathbf{x})\rho(\mathbf{x}|\mathbf{z})P(\mathbf{z})d\mathbf{z} \right)^2 d\mathbf{x}, \tag{21}$$

over A, subject to positivity (and possibly regularity) constraints.

Additionally, it has been shown that the attractiveness field is nonstationary over time scales of several days to several months, stemming from self-exciting effects of recent criminal activity [12, 17, 18]. Here the estimated marginal density of crime $\hat{P}_c(\mathbf{x})$ could be reconstructed at the time of an offense using self-exciting point processes fit to crime data [12] from the preceding several weeks or months.

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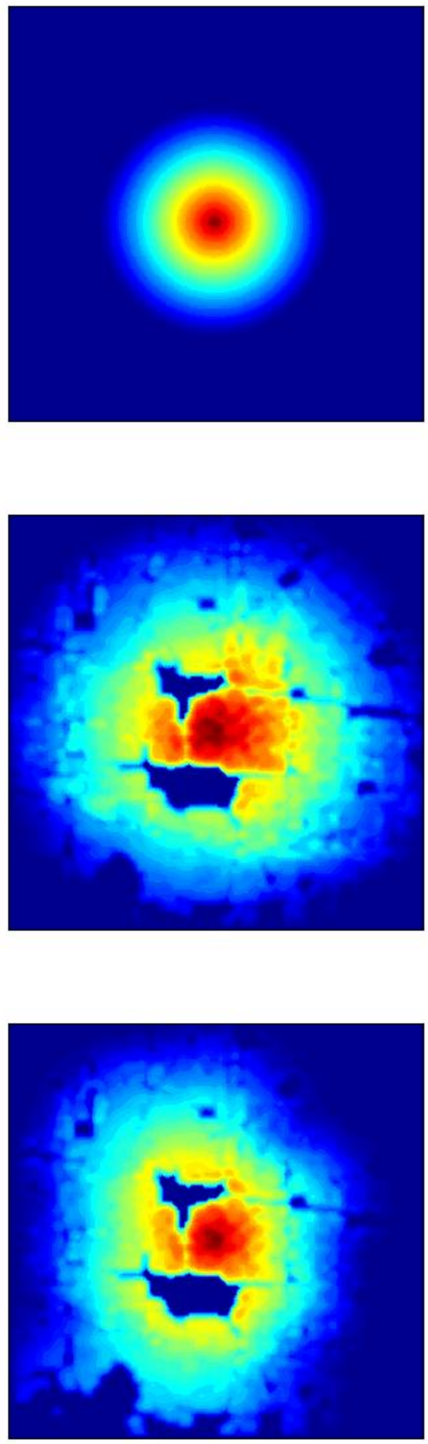

Figure 3: Geographic profiles (plotted on a logarithmic scale) for one crime using model 1 (top), model 2 (center), and model 3 (bottom) with best fit parameters.

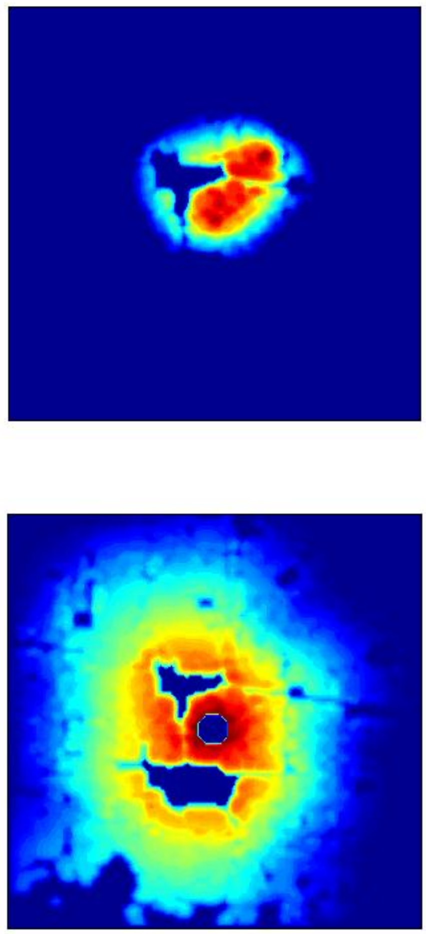

Figure 4: Geographic profiles (plotted on a logarithmic scale) for two crimes (top) and one crime with a buffer zone (bottom) using model 3 with best fit parameters.

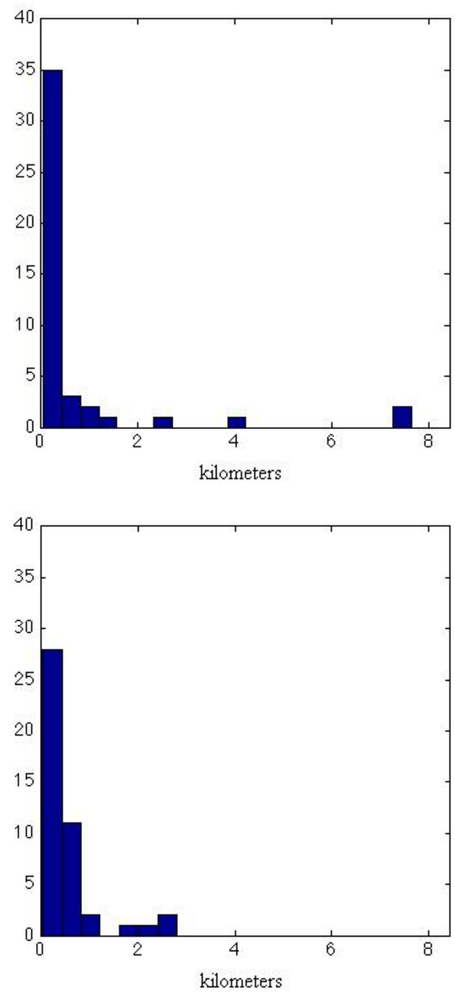

Figure 5: Histogram of distances to crime for 2004 data (top) and simulated data using model 3 with best fit parameters (bottom).