Gene Kim
Math 164
Homework 3

4.2.2. First, we make the substitutions $x_1 = x_1' - 2$, $x_2 = x_2' - x_2''$, $x_3 = x_3 - x_3''$, and $\hat{z} = z + 2$. Then, we add an excess variable and a slack variable to get:
minimize $\hat{z} = x_1' - 5x_2' + 5x_2'' - 7x_3' + 7x_3''$
subject to

$$\begin{array}{rcl} 5x_1' - 2x_2' + 2x_2'' + 6x_3' - 6x_3'' - e_1 & = & 15 \\ 3x_1' + 4x_2' - 4x_2'' - 9x_3' + 9x_3'' & = & 9 \\ 7x_1' + 3x_2' - 3x_2'' + 5x_3' - 5x_3'' + s_1 & = & 23 \\ x_1', x_2', x_2'', x_3', x_3'', s_1, e_1 \geq 0 & & \end{array}.$$

4.3.4. (b) $(0,0), (0,12), (6,24)$
(c) $(1,0)$ and $(2,3)$ are linearly independent directions of unboundedness.
(d) You can obtain the same results by using standard form.

4.4.3. We have $x = \sum_{i=1}^{k} \alpha_i y_i$, where $\sum_i \alpha_i = 1$ and $\alpha_i \geq 0$ for all i and $y_i = \sum_{j=1}^{k_i} \beta_{ij} y_{ij}$, where $\sum_j \beta_{ij} = 1$ and $\beta_{ij} \geq 0$ for all i, j. Then,

$$x = \sum_{i=1}^{k} \alpha_i \sum_{j=1}^{k_i} \beta_{ij} y_{ij} = \sum_{i,j} \alpha_i \beta_{ij} y_{ij},$$

where

$$\sum_{i,j} \alpha_i \beta_{ij} = \sum_{i=1}^{k} \alpha_i \left(\sum_{j=1}^{k_i} \beta_{ij} \right) = \sum_{i=1}^{k} \alpha_i = 1.$$

Thus, x is a convex combination of the vectors $\{y_{ij}\}$.

4.4.5. We have that

$$Ad = A \left(\sum_{i=1}^{k} \alpha_i d_i \right) = \sum_{i=1}^{k} \alpha_i (Ad_i) = 0,$$

and we know that d is nonnegative since it is a nonnegative linear combination of nonnegative vectors. Hence, it follows that d is a direction of unboundedness.