

MATH 32A: REVIEW PROBLEMS FOR MIDTERM 1

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Problem 1. Find the equation of the plane through $(1, 1, 1)$ that contains the line

$$x = t + 1, \quad y = 2 - 3t, \quad z = 5 - 2t.$$

Problem 2. Find the distance between two lines L_1 and L_2 :

$$\begin{array}{lll} L_1 : x = 1 + t & y = 5 + 2t & z = -t \\ L_2 : x = 3s & y = 2 - s & z = -1 + 4s. \end{array}$$

Problem 3. Find the equation of the line of intersection of the two planes $x + y + z = 6$ and $2x - y + 3z = 6$.

Problem 4. Describe the set of points P such that $|\overline{PA}| = 2|\overline{PB}|$, where $A = (1, -4, -2)$ and $B = (-2, -4, 1)$.

Problem 5. Find the two points of intersection of the two space curves

$$\vec{r}_1(t) = \langle t, t^2, t^3 \rangle \quad \text{and} \quad \vec{r}_2(s) = \langle 1 + 2s, 1 + 6s, 1 + 14s \rangle.$$

Find the angles of the intersection at the points.

Problem 6. Reparametrize the curve

$$\vec{r}(t) = \langle 2e^t \cos(t), 2e^t \sin(t), e^t \rangle$$

with respect to arc length and find the curvature at $t = 0$.

Problem 7. Find \vec{T} , \vec{N} , \vec{B} , the normal and osculating plane of the curve at the given point:

$$\vec{r}(t) = \langle 2e^t \cos(t), 2e^t \sin(t), e^t \rangle \quad \text{at} \quad (2, 0, 1).$$

Problem 8. Let $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$. Show that $\vec{r} \perp \vec{r}'$.

Problem 9. If $\vec{r} = \langle x, y \rangle$, $\vec{a} = \langle a_1, a_2 \rangle$, and $\vec{b} = \langle b_1, b_2 \rangle$, show that the vector equation $(\vec{r} - \vec{a}) \cdot (\vec{r} - \vec{b}) = 0$ represents a circle, and find its center and radius.

Problem 10.

(a) Show that if \vec{a} , \vec{b} , \vec{c} are vectors, then

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c}).$$

(b) Suppose \vec{w} , \vec{u} , \vec{v} are differentiable vector functions. Find the derivatives of

$$\vec{u} \cdot (\vec{v} \times \vec{w}) \quad \text{and} \quad (\vec{u} \times \vec{v}) \cdot \vec{w}$$

and show that they are the same.