Assignment 3

1. Show that if $f \in L^2([-\pi,\pi], \mathcal{B}, \frac{\lambda}{2\pi})$, and
$$f(x) \sim \sum a_n e^{inx},$$
then $|a_n| \to 0$ as $n \to +\infty$ and $n \to -\infty$.
[Don't use the more general result for $f \in L^1$]

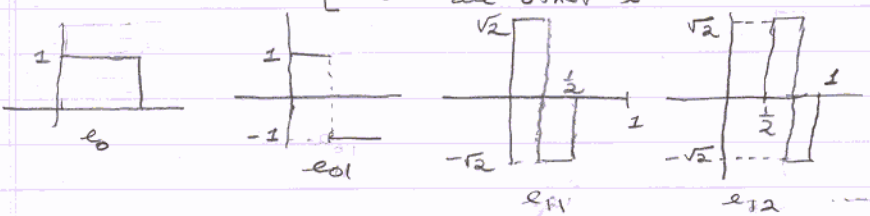
2. page 68: 46, 50, 51

3. Show that the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1$ is $\pi a b$ by performing a change of variables (you can assume that the unit circle has area 1)

4. Define the Haar functions on $[0,1]$ by

$$e_0 = 1 \qquad 0 \le x \le 1$$

$$\boxed{\begin{array}{l} 0 \le m \\ k \le 2^m \end{array}} \qquad e_{mk}(x) = \begin{cases} 2^{m/2} & \frac{k-1}{2^m} \le x < \frac{k-\frac{1}{2}}{2^m} \\ -2^{m/2} & \frac{k-\frac{1}{2}}{2^m} \le x < \frac{k}{2^m} \\ 0 & \text{all other } x \end{cases}$$



a) Show that these functions are orthonormal in $L^2([0,1], \mathcal{B}, \lambda)$ [Note: if $m < n$, then $e_{nk} \neq 0$ on a set where $e_{m\ell}$ is constant]

b) Show they form a basis.

Hint: Say that $f \perp e_0, e_{01}, e_{11}, e_{12}, e_{21}, e_{22}, e_{23}, e_{24}$ ...
You can use the fact that $F(x) = \int_0^x f(t)\,dt$ is continuous and $F'(x) = f(x)$ [we'll prove this later]
Successively use $f \cdot e_0 = 0 \Rightarrow F(1) - F(0) = 0 \Rightarrow F(1) = F(0) = 0$
$f \cdot e_{01} = 0 \Rightarrow F(\frac{1}{2}) = 0, \ldots \Rightarrow F = 0.$