

**Math 131A/2 Winter 2002 Handout #1a**

**Instructor:** E. Effros, MS 6931.

**Lecture Meeting Time:** MWF 2:00PM-2:50PM **Location:** MS 5117

Recitation TA: to be announced, MS 5117 T 2:00P-2:50P

**Office hours** (tentative): TF 4-5.

**Text** Fundamental Ideas of Analysis by Michael Reed

**Grading system:** class quizzes: 10%, homework: 10%, hour exams: 20% each, final:40%

**Hour Exam Dates:** February 4, February 22

There are no make-up exams. If you have an excused absence, your grade will be based on the other exams and homework.

Homework will be collected in lectures (beginning the second week of the term), and will be assigned roughly each week.

About this course: This is perhaps the most important undergraduate mathematics course that we offer. In many ways it serves as the gateway to graduate mathematics. Although it is concerned with a specific mathematical discipline, analysis, its most important function is to train students how to read and write mathematics.

This may be the most demanding math course that you will take as an undergraduate. Try not to get discouraged and remember that you are not alone! In order to keep up it will be absolutely critical that you :

a) Attend all lectures and take notes - it would be best if you reviewed your previous notes before you come to lecture.

b) Read the book and handouts with the utmost care. Don't be surprised if you find yourself spending an hour understanding a single page!!

c) Carefully do the problems (don't just copy them from someone else!) Keep in mind that this we will be much more concerned with theorems and concepts than with computations. On homework and tests: Don't write answers without a justification.

d) Frequently go to the webpage for assignments, corrections, announcements etc.

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Assignment 1

1. You plan to spend the night at a hotel with infinitely many rooms. You are told that all the rooms are full. Why is it that they can put you up for the night anyway? Use complete sentences in your explanation.

2. a) You place 100 balls on the ground at 11:00AM and remove the first. You place 100 more balls on the ground at 11:30, and remove the second ball. You replace 100 more balls at 11:45, and remove the third ball. Etc. How many balls will be there at 12 noon? Use complete sentences in your explanation. b) What would happen if you instead removed the 101st ball at time 11:30, the 201st ball at 11:45, etc.?

3. 1.1: 8, 9,10

4. 1.2: 1, 2, 4, 5, 7, 10

5. State the denials of the following statements (dont just insert the words "It is not true that ...")
- Either you take me for a fool, or you must be a fool yourself.
  - He walked into my office this morning, told me a pack of lies, and punched me on the nose.
  - If you are right, then I am wrong.
  - You are right if and only if I am wrong.

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**Some pointers on the logical conventions of mathematics.**

It is probably hardest to get used to the way mathematicians use the word "implies" or the symbol  $\Rightarrow$ . The idea is that you want to know if the implication is true or not just on the basis of whether the ingredients are true or false **without any more thinking**.

- Any true statement  $\Rightarrow$  any true statement (e.g.,  $1 + 1 = 2 \Rightarrow$  there are infinitely many primes  $\Rightarrow$  Fermat's last conjecture)
- Any false statment  $\Rightarrow$  any true statement (because, in particular, you want to be able to say that  $1 = 2 \Rightarrow 1 \times 0 = 2 \times 0$  is a correct "deduction").
- Any false statement  $\Rightarrow$  any false statement (because, in particular, you want to be able to say that  $1=2 \Rightarrow 1 + 1 = 2 + 1$  is a correct "deduction").
- The following is **false**: truth  $\Rightarrow$  false

1. The most common logical **errors** made by beginners:

- They think that "or" is exclusive: thus although they know that  $\leq$  means "less than or equal to" they think it is "wrong" to write  $3 \leq 3$  because they know that "actually,  $3 = 3$ "
- They think that you cannot prove  $P \Rightarrow Q$  if you already know that  $P$  is false.
- They think that if  $P \Rightarrow Q$  is true, then  $Q$  is true.
- When asked to prove  $P \Rightarrow Q$  they instead prove  $Q \Rightarrow P$ .
- They get  $=$  (equality - say of numbers) mixed up with  $\Leftrightarrow$  (logical equivalence, used for propositions).

2. Some *correct* illustrations of logic:

- The proposition " $6 < 7$  or  $4 < 5$ " is true.
- The proposition " $1 = 2 \Rightarrow 0 \times 1 = 0 \times 2$ " is true.
- The proposition "For any real number  $x$ ,  $x + 2 = 3 \Rightarrow x(x + 2) = x3$ " is true.

- The proposition “For any real number  $x$ ,  $x(x+2) = x3 \Rightarrow x+2 = 3$ ” is false.

3. It is important to be able to take the negations of statements (in order to prove things by contradiction). Here is the general scheme (don't worry about the last two at this point)

$$\begin{aligned}\sim(P \text{ or } Q) &\Leftrightarrow (\sim P) \text{ and } (\sim Q) \\ \sim(P \text{ and } Q) &\Leftrightarrow (\sim P) \text{ or } (\sim Q) \\ \sim(P \Rightarrow Q) &\Leftrightarrow (P \text{ and } \sim Q) \\ \sim((\forall x)P(x)) &\Leftrightarrow (\exists x)(\sim P(x)) \\ \sim((\exists x)P(x)) &\Leftrightarrow (\forall x)(\sim P(x))\end{aligned}$$