Math 33A Final Review

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1 Overview

The final exam is cumulative and will cover all the material in the lectures. Your review should, therefore, include the information on the study guides for each of the first two midterms. Also chapter 7 and crucial ideas from chapter 8 will be fair game.

1.

2 Practice problems

The actual final exam will consist of problems that are related to the ones below. The structure will be similar to that of the two midterms.

1. As a warm-up you should look at the true/false questions on pages 362-363 and 391-392.
2. The following question is review from the first part of the course, and is foundational:
   (a) Use Gauss-Jordan elimination to solve the following system of linear equations. Indicate whether there are zero, one, or infinitely many solutions.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + x_4 &= 1 \\ 2x_1 + x_2 - x_3 + 4x_4 &= 2 \\ 7x_1 + 5x_2 + 13x_4 &= 7 \end{aligned}$$

   (b) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & \sqrt{3}/2 \\ 0 & \sqrt{3}/2 & -1/2 \end{bmatrix}$$

   (c) Describe the matrix in the second part geometrically.
3. For the following matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 1 & 2 \\ 1 & -2 & 3 & 1 \\ 1 & -1 & 2 & 1 \end{bmatrix}$$

(a) Find rref(A).

(b) Find a basis for im(A)

(c) Find a basis for ker(A)

4. Consider the system $A\vec{x} = \vec{0}$ where

$$A = \begin{bmatrix} 1 & 1 & t \\ -1 & t & 1 \\ t & 1 & 1 \end{bmatrix}$$

where t is any real number.

(a) For which values of t is the kernel of A not equal to $\{\vec{0}\}$

(b) For each value of t which makes the kernel of A not equal to $\{\vec{0}\}$ find a basis for the kernel of A.

5. The following problem is about QR-Decomposition. You should be comfortable computing this. Also you should understand the relationship between this process, orthogonal matrices, and projection matrices.

(a) Find the QR-decomposition of the following matrix, whose columns are linearly independent:

$$A = \begin{bmatrix} 0 & -3 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

(b) Find a matrix which represents orthogonal projection onto the span of the columns of Q. (Hint: the columns of Q are orthonormal if you did part (a) correctly.)

(c) Find the volume of the parallelopiped spanned by the columns of A

6. Use the method of least squares approximation to find the equation of the line which best fits the following points in the xy-plane: $(-4, -2), (-1, 0), (1, 2), (2, 3)$. When you've found the line, sketch its graph in the xy-plane along with the four original data points.

7. Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 4 \\ 0 & 1 & 1 \end{bmatrix}$$

and

$$\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

.

(a) For the equation $A\vec{x} = \vec{b}$, find an equation for all the least-squares solutions, $\vec{x}^*$.

(b) Of the solutions you found in the first part, find the solution which is the shortest.

8. For the symmetric matrix

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

   (a) Find its eigenvalues.

   (b) Find an orthonormal eigenbasis

   (c) Compute A^4

9. For the matrix

$$A = \begin{bmatrix} 4 & -2 \\ 5 & 6 \end{bmatrix}$$

   (a) find its complex eigenvalues.

   (b) find the corresponding complex eigenvectors

   (c) Find matrices S and B such that $A = SBS^{-1}$ where B is a rotation-dilation matrix.

10. For the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

   (a) find its singular values.

   (b) find its singular value decomposition.