1. Consider the functor \( F : \text{Ab} \to \text{Ab} \) defined by \( F(A) = A \oplus \mathbb{Z} \) and \( F(f) = f \oplus \text{id}_\mathbb{Z} \) for \( f : A \to B \). Determine whether or not this functor is faithful and whether or not it is full.

2. Let \( \mathcal{C} \) be a category, and define a skeleton of \( \mathcal{C} \) to be a full subcategory which contains a unique representative of the isomorphism class of each object. Show that
   a. If \( \mathcal{S} \) is a skeleton of \( \mathcal{C} \), then the inclusion functor \( \mathcal{S} \to \mathcal{C} \) is an equivalence of categories.
   b. Two categories \( \mathcal{C} \) and \( \mathcal{D} \) with skeletons \( \mathcal{S} \) and \( \mathcal{T} \), respectively, are equivalent if and only if \( \mathcal{S} \) and \( \mathcal{T} \) are isomorphic.

3. Prove that \( \text{Set} \) is not equivalent to its opposite category.

4. Let \( n \geq 2 \).
   a. Find the sequential limit in \( \text{Ab} \) of the diagram
      \[
      \cdots \to \mathbb{Z} \overset{n}{\to} \mathbb{Z} \overset{n}{\to} \mathbb{Z} \overset{n}{\to} \mathbb{Z}.
      \]
   b. Repeat part a with \( \mathbb{Z} \) replaced by \( \mathbb{Q} \).

5. Show that
   \[
   (A_1 \times A_2) \times A_3 \cong \prod_{i=1}^{3} A_i
   \]
   for \( A_1, A_2, A_3 \) objects in any category \( \mathcal{C} \) that admits finite products.

6. What are the initial and terminal objects in the category \( \text{Rings} \) of rings (with unit)? Prove that your answer is correct.

7. Let
   \[
   \begin{array}{ccc}
   A & \xrightarrow{p_1} & B_1 \\
   p_2 \downarrow & & \downarrow f_1 \\
   B_2 & \xrightarrow{f_2} & C
   \end{array}
   \]
   be a pullback diagram in a category \( \mathcal{C} \) (in the sense that \( A \cong B_1 \times_{\mathcal{C}} B_2 \)). Prove or provide a counterexample to each of the following:
   a. If \( f_1 \) is a monomorphism, then so is \( p_2 \).
   b. If \( f_1 \) is an epimorphism, then so is \( p_2 \).
8. An equalizer \( \text{eq}(f, g) \) of morphisms \( f, g: A \to B \) in a category \( C \) is a limit of the diagram

\[
A \xrightarrow{f} B.
\]

Suppose that \( C \) has equalizers of every pair of morphisms. Let \( F: C \to D \) be a functor such that \( F \) preserves equalizers (i.e., if \( \iota: \text{eq}(f, g) \to A \) is the morphism given by definition of a limit, which is to say that \( \iota \) equalizes \( f \) and \( g \), then \( F(\iota) \) is the morphism that equalizes \( F(f) \) and \( F(g) \)) and which is such that if \( F(f) \) is an isomorphism, then \( f \) is. Show that \( F \) is faithful.