1. **5 points** Sketch the domain $D$ bounded by $y = x^2$, $y = \frac{1}{2}x^2$ and $y = x$. Use the change of variables: $x = uv$ and $y = u^2$ to compute $\int \int_D \frac{1}{y} \, dx \, dy$.

$\chi = uv, \quad y = u^2$

Here $x > 0$, $y > 0$, since $D$ is in the first quadrant.

$\therefore uv > 0 \Rightarrow u > 0 \quad \text{or} \quad v > 0$

We will choose $u > 0$ and $v > 0$.

Now, $y = x^2 \Rightarrow u^2 = (uv)^2$

$\Rightarrow u^2 = uv$

$\Rightarrow 1 - v^2$

$\Rightarrow v = \sqrt{1}$

$y = \frac{1}{2}x^2 \Rightarrow u^2 = \frac{1}{2} (uv)^2 \Rightarrow v = \sqrt{2}$

$y = x \Rightarrow u^2 = uv \Rightarrow u = v$ or $u = 0$

$\frac{\partial (x, y)}{\partial (u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2u & 0 \\ 0 & 2v \end{vmatrix} = 4uv$

$\left| \frac{\partial (x, y)}{\partial (u, v)} \right| = \left| -2u^2 \right| = 2u^2$

\[ \int \int_D \frac{1}{y} \, dx \, dy = \int_{u=0}^{\sqrt{2}} \int_{v=0}^{\sqrt{2}} \frac{1}{u^2} \cdot 2uv \, du \, dv = \int_{v=0}^{\sqrt{2}} 2v \, dv = v^2 \bigg|_{v=0}^{v=\sqrt{2}} = 2 - 0 = 2 \]
2. **5 points** Use spherical coordinates to find the volume of the region bounded below by the plane $z = 1$ and above by the sphere $x^2 + y^2 + z^2 = 4$.

\[
\begin{align*}
\chi &= \rho \sin \phi \cos \theta \\
y &= \rho \sin \phi \sin \theta \\
z &= \rho \cos \phi \\
2 &= 1 \Rightarrow \rho \cos \phi = 1 \\
\Rightarrow \rho = \frac{1}{\cos \phi} = \sec \phi
\end{align*}
\]

The required volume equals the volume of the upper half of the sphere above the plane $z = 1$.

\[
\begin{align*}
0 &\leq \phi \leq \frac{\pi}{2} \\
0 &\leq \rho \leq \sec \phi
\end{align*}
\]

We need to find the limits of $\phi$.

\[x^2 + y^2 + 2^2 = 4 \]

\[\Rightarrow \rho^2 = 4 \]

\[\Rightarrow \sec^2 \phi = 4 \]

\[\Rightarrow \cos \phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}, \frac{2\pi}{3}. \text{ From our picture we see that } \phi = \frac{\pi}{3} \]

\[
\begin{align*}
\text{Volume} &= \int_{0}^{\frac{\pi}{2}} \int_{0}^{\frac{\pi}{3}} \int_{0}^{\rho} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \\
&= \frac{50\pi}{3}
\end{align*}
\]
3. **5 points** Evaluate the vector line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F} = (z^3, yz, x) \) and \( C \) is the quarter of the circle of radius 2 in the \( yz \)-plane with center at the origin where \( y \geq 0 \) and \( z \geq 0 \), oriented clockwise when viewed from the positive \( x \)-axis.

\[
\mathbf{r}(t) = \langle 0, \sqrt{2} \cos t, \sqrt{2} \sin t \rangle, \quad 0 \leq t \leq \frac{\pi}{2}
\]

\( y = \sqrt{2} \cos t, \quad z = \sqrt{2} \sin t \)

\[
\mathbf{F}(\mathbf{r}(t)) = \langle 2\sqrt{2} \sin^3 t, 2\sqrt{2} t \sin^2 t, 0 \rangle
\]

\[
\mathbf{F}(\mathbf{r}'(t)) \cdot \mathbf{r}''(t) = -2\sqrt{2} \cos^3 t \sin^2 t
\]

\[
\int_C \mathbf{F} \cdot d\mathbf{r} = - \int_0^{\pi/2} (-2\sqrt{2} \cos^3 t \sin^2 t) \, dt
\]

= Complete the integration.
4. **5 points** Is the vector field \( \mathbf{F} = (e^z(z + 1), -\cos y, e^z) \) conservative? If so, find its potential function.

The domain of \( \mathbf{F} \) is the entire \( \mathbb{R}^2 \), which is simply connected. So \( \mathbf{F} \) will be conservative if \( \text{curl}(\mathbf{F}) = 0 \).

\[
\text{curl}(\mathbf{F}) = \begin{vmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
0 & 0 & \frac{\partial}{\partial z} \\
e^{x+1} & -\cos y & 0
\end{vmatrix}
\]

\[
= \hat{k} (0 - 0) + \hat{j} (e^z - e^z) + \hat{i} (0 - 0)
\]

\[= 0 \]

\( \mathbf{F} \) is a conservative vector field.

From simple observation we see that

\[ f(x, y, z) = e^z(x + 1) + \sin y + e^z + \zeta \]

are all the potential functions.

\( \zeta \) = constant.