Solutions for More (Practice Problems) for 2nd Midterm

1. It is easy to show that multiplication is represented in Q by $v_1 \cdot v_2 = v_3$. (See the proof of Lemma 5.11.) It follows that the function $b \mapsto b \cdot b$ is represented in Q by $v_1 \cdot v_1 = v_2$. Since a is a square $\Leftrightarrow \exists b(b < S(a) \wedge b \cdot b = a)$, closure under bounded quantification implies that the set of squares is represented in Q.

2. $\exists v_7(v_7 \boldsymbol{<} \mathbf{S} v_1 \wedge v_7 \cdot v_7 = v_1)$ is such a formula.

3. Define $g_2' : \mathbb{N}^2 \to \mathbb{N}$ by setting $g_2'(a_1, a_2) = g_2(I_2^2(a_1, a_2))$. The function g_2' is primitive recursive by closure under Compostion. Let $g_3 : \mathbb{N}^2 \to \mathbb{N}$ be the constant function with value 5. Thus

$$h(a_1, a_2) = f(g_1(a_1, a_2), g_2'(a_1, a_2), g_3(a_1, a_2)).$$