

## Sample Midterm Math 31A

Student ID : \_\_\_\_\_

First Name: \_\_\_\_\_

Last Name: \_\_\_\_\_

There are a total of 5 problems. SHOW YOUR WORK ON ALL PROBLEMS. Please write clearly.

1. Find all the local maxima and local minima of the function

$$f(x) = x^{\frac{2}{3}}(1-x)^2.$$

Justify your answer.

2. Consider the function

$$f(x) = \frac{x^2}{\sqrt{x^4 - 1}}.$$

- (i) Find the domain of  $f$ .
- (ii) Find the horizontal and vertical asymptotes of  $f(x)$ , if there are any.

Justify your answer.

3. Prove that

$$\frac{1}{3} \leq \int_4^6 \frac{1}{x} dx \leq \frac{1}{2}.$$

Justify your answer.

4. Consider the point  $P = (5, 3)$  in the plane.

- (i) Find the slope of the line through  $P$  such that the triangle bounded by this line and the axes in the first quadrant has minimal area.
- (ii) Show that  $P$  is the midpoint of the hypotenuse of this triangle.

Justify your answer.

5. Consider the function  $f(x) = x^2(x - 4)$ . Determine the intervals on which the function is concave up or down and find the points of inflection.

Justify your answer.