

Student ID : \_\_\_\_\_

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There are a total of 4 problems. SHOW YOUR WORK ON ALL PROBLEMS. Please write clearly.

1. Give examples of two sequences  $(a_n)_{n=1}^{\infty}$  and  $(b_n)_{n=1}^{\infty}$  which are different (as functions from  $\mathbb{N}$  to  $\mathbb{R}$ ), but such that each is a subsequence of the other.
2. Let  $f(x) = \sin(\frac{1}{x})$  for  $x \neq 0$  and let  $f(0) = 0$ .
  - (i) Show that  $f$  is discontinuous at 0.
  - (ii) Show that  $f$  has the intermediate value property on  $\mathbb{R}$ .
3.
  - (i) Let  $S \subset \mathbb{R}$ ,  $f : S \rightarrow \mathbb{R}$  be a continuous function. Let  $(s_n)_{n=1}^{\infty}$  be a Cauchy sequence in  $S$ . Prove that  $(f(s_n))_{n=1}^{\infty}$  is also a Cauchy sequence.
  - (ii) Let  $f : (0,1) \rightarrow \mathbb{R}$  defined by  $f(x) = \frac{1}{x^{10}}$ . Show that  $f$  is not uniformly continuous.
4. Show that there is no continuous function  $f : \mathbb{R} \rightarrow \mathbb{R}$  with the property that  $(f \circ f)(x) = -x$  for every  $x \in \mathbb{R}$ .