

Section: \_\_\_\_\_

First Name: \_\_\_\_\_

Last Name: \_\_\_\_\_

There are a total of 5 problems. SHOW YOUR WORK ON ALL PROBLEMS. Please write clearly.

1. Prove that for all  $n \in \mathbb{N}$

$$1 + 3 + 5 + \dots + (2n - 1) = n^2.$$

2. Let  $S$  be a set bounded from below. Then  $s_0 = \inf S$  if and only if:

(i)  $s_0 \leq x$ , for all  $x \in S$ .

(ii) for any  $\varepsilon > 0$ , there exists  $x \in S$  such that  $x < s_0 + \varepsilon$ .

3. Let  $A$  and  $B$  be nonempty bounded below subsets of  $\mathbb{R}$ , and let  $S := \{a + b : a \in A, b \in B\}$ . Prove that

$$\inf S = \inf A + \inf B.$$

4. Let  $(a_n)$  be a sequence which converges to  $a \in \mathbb{R}$ . Show that if  $a_n \neq 0$  for all  $n$ , and if  $a \neq 0$ , then  $(\frac{1}{a_n})$  converges to  $\frac{1}{a}$ . (**Hint:** Show first that there is  $m > 0$  such that  $|a_n| \geq m$  for all  $n \in \mathbb{N}$ ).

5. Compute

$$\lim_{n \rightarrow \infty} \frac{2n^{2008} + n^{2007} + 3}{4n^{2008} + 2007n^{2006} - 12}.$$