

## Sample Midterm Math 31A

Student ID : \_\_\_\_\_

First Name: \_\_\_\_\_

Last Name: \_\_\_\_\_

There are a total of 5 problems. SHOW YOUR WORK ON ALL PROBLEMS. Please write clearly.

1. A conical tank has height 3 m and radius 2 m at the top. Water flows in at a rate of  $2 \text{ m}^3/\text{min}$ . How fast is the water level rising when it is 2 m?
2. Find the points on the graph of  $x^3 - y^3 = 3xy - 3$  where the tangent line is horizontal.
3. Let  $f$  be the function defined by  $f(x) = |x^2 - 1|$ . Find the points  $c$  (if any) such that  $f'(c)$  does not exist.
4. (i) Find numbers  $a$  and  $b$  such that  $\lim_{x \rightarrow 0} \frac{\sqrt{ax+b}-2}{x} = 1$ .  
(ii) Evaluate the limit  $\lim_{x \rightarrow 0} x \left[ \frac{1}{x} \right]$ , where  $[x]$  denotes the greatest integer function.
5. Suppose that  $f$  and  $g$  are differentiable functions such that  $f(g(x)) = x$  and  $f'(x) = 1 + [f(x)]^2$ . Show that  $g'(x) = \frac{1}{1+x^2}$ .