

Math 115AH: Problem Set 5

due Friday, November 6

Read: Sections 5.1 and 5.2. Also look over Chapter 4 (this should mostly be review from Math 33A).

1. Problems 12, 20, 21 from Section 4.3.
2. Problems 4(e), 14, 17 from Section 5.1.
3. Problems 2(a), 2(b), 3(b), 8 and 12 from Section 5.2.
4. Let $a, b, r_1, \dots, r_n \in \mathbb{R}$. Set

$$f(t) = (r_1 - t)(r_2 - t) \dots (r_n - t),$$

and consider the determinant

$$D(x) = \det \begin{pmatrix} x + r_1 & x + a & x + a & \dots & x + a \\ x + b & x + r_2 & x + a & \dots & x + a \\ x + b & x + b & x + r_3 & \dots & x + a \\ \dots & \dots & \dots & \dots & \dots \\ x + b & x + b & x + b & \dots & x + r_n \end{pmatrix}$$

- (a) Show that $D(x)$ is a polynomial in x of degree at most one. (*Hint:* Subtract the first column from all other columns.)
- (b) Calculate $D(-a)$ and $D(-b)$.
- (c) Assuming that $a \neq b$, deduce that

$$D(0) = \frac{bf(a) - af(b)}{b - a}.$$