Math 115AH: Problem Set 2
due Friday, October 9

Read: Sections 1.1-1.4.

1. Fill in the blanks in the proof of the following Theorem. You can write directly on this sheet, and attach it to your homework.

Theorem 1. *Let X, Y and Z be sets. If $f : X \to Y$ and $g : Y \to Z$ are both injective, then $g \circ f : X \to Z$ is injective.*

Proof. Suppose that $x_1, x_2 \in X$ are such that $(g \circ f)(x_1) = (g \circ f)(x_2)$. We will show that _____. Let $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Then $g(y_1) =$ _____. So, since g is injective, _____. Therefore, $f(x_1) =$ _____. So, since _____, we get that $x_1 = x_2$, as desired. □

2. Do problems 8 and 18 from Section 1.2. For problem 8, write exactly which of the vector space axioms you are using.

3. (a) Give an example of a subset $W \subseteq \mathbb{R}^2$ which is closed under scalar multiplication, but not under vector addition.

(b) Give an example of a subset $W \subseteq \mathbb{R}^2$ which is closed under vector addition, but not under scalar multiplication.

4. Which of the following subsets W are subspaces of $C[0, 1]$? You must always jusstify your answers.

(a) $W = \{f \in C[0, 1] \mid f(0) = f(1)\}$
(b) $W = \{f \in C[0, 1] \mid f(0) = 0 \text{ and } f(1) = 0\}$
(c) $W = \{f \in C[0, 1] \mid f(0) = 0 \text{ or } f(1) = 0\}$
(d) $W = \{f \in C[0, 1] \mid f \text{ is nondecreasing }\}$

5. Do problems 26 and 31 from Section 1.3.

6. Do problems 10 and 15 from Section 1.4.