

**Math 115AH: Problem Set 1**  
due Friday, October 2

Read: Chapter 2 in the Supplement (pages 605-624), and Appendices A, B, C (pages 549-555).

1. Do exercises 2.4, 2.48, 2.50 and 2.53 from the Supplement to the textbook, pages 624-629.

2. Let  $\mathbb{F}$  be a field and  $M_n(\mathbb{F})$  the set of  $n$ -by- $n$  matrices with entries in  $\mathbb{F}$ , with the usual multiplication and addition of matrices. Let  $0$  be the zero matrix and  $1$  the identity matrix. Show that  $M_n(\mathbb{F})$  is a field if and only if  $n = 1$ .

3. Consider the set  $A \subset M_2(\mathbb{R})$  of matrices of the form  $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ , where  $a$  and  $b$  are real numbers. Show that  $A$  is a field.

4. Construct a field with exactly three elements, and a field with exactly four elements.