Math 225A: Problem Set 5

due Friday, December 4

1. Guillemin and Pollack Ch.3 §2: problems 6, 18, 25 on p.105-107.

2. Guillemin and Pollack Ch.3 §3: problems 18, 19, 20 on p.118-119.

3. Guillemin and Pollack Ch.3 §4: problems 1, 7, 8, 10(a), 11 on p.130-132.

4. Guillemin and Pollack Ch.3 §7: problem 3 on p.150.

5. Read the statements of exercises 11-16 from Guillemin and Pollack Ch.3 §5 on p.140-141. You do not have to write down solutions for these (though I encourage you to work on them on your own, if you have time).

(a) Assuming the conclusions of those exercises, write up a short solution for problem 17 on p.141.

(b) Let Σ_g be the genus g surface in $\mathbb{R}^3$ given by the equation

$$z^2 = (4(g+1)^2 - x^2 - y^2) \prod_{m=0}^{g-1} ((x - 2(g-1) + 4m)^2 + y^2 - 1),$$

where $g \geq 0$. Show that x is a Morse function on Σ_g, find its critical points and their indices, and then compute $\chi(\Sigma_g)$ using problem 17 above.

(c) Let $X = \mathbb{RP}^n$ be the real projective space of dimension n, with homogeneous coordinates $x_0, \ldots, x_n$. Define a function

$$f : \mathbb{RP}^n \to \mathbb{R}, \quad f([x_0 : x_1 : \cdots : x_n]) = \frac{\sum_{m=0}^n m|x_m|^2}{\sum_{m=0}^n |x_m|^2}.$$

Show that f is well-defined and Morse, and use it to compute $\chi(\mathbb{RP}^n)$.