Math 225A: Problem Set 4

due Wednesday, November 25

1. Guillemin and Pollack Ch.2 §1: problem 3 on p.62.

2. Guillemin and Pollack Ch.2 §2: problem 7 on p.66.

3. Guillemin and Pollack Ch.2 §3: problems 1, 2, 3, 5, 12, 14, 16, 20 on p.74-77.

4. Guillemin and Pollack Ch.2 §4: problems 2, 3, 10, 18 on p.82-84.

5. Guillemin and Pollack Ch.2 §5: problem 7 on p.88.

6. Guillemin and Pollack Ch.2 §6: problem 3 on p.93.

7. Let $f : \mathbb{R}^{n+1} \to \mathbb{R}$ be a smooth function. Suppose that $0 \in \mathbb{R}$ is a regular value, and let X be the smooth manifold $f^{-1}(\{0\})$.

(a) Show that X has a non-vanishing *normal field*, i.e. a smooth section of the normal bundle of X in $\mathbb{R}^{n+1}$. In other words, you are asked to show that there exists a smooth map $N : X \to \mathbb{R}^{n+1}$ such that $N(x) \neq 0$ for any $x \in X$, and $\langle N(x), v \rangle = 0$ for any $x \in M$ and $v \in T_xX \subset T_x\mathbb{R}^{n+1} \cong \mathbb{R}^{n+1}$. Here, $\langle \cdot, \cdot \rangle$ denotes the standard inner product in $\mathbb{R}^{n+1}$.

(b) A smooth manifold Y of dimension n is called *parallelizable* if there exist n vector fields $v^1, \ldots, v^n$ on Y such that at every $y \in Y$, the tangent vectors $v^1_y, \ldots, v^n_y$ are linearly independent. Use (a) to show that $X \times S^1$ is parallelizable. (In particular, $S^n \times S^1$ is parallelizable.)