Math 212 **Homework 3** **Due March 2nd, 2009**

1. Let $\mathcal{A}$ be an abelian category with enough injectives and enough projectives. Show that $\mathrm{Ext}_{\mathcal{A}}$ is balanced; that is, given objects A and B, an injective resolution $B \to I^\bullet$ and a projective resolution $P_\bullet \to A$, show that

$$H^*\mathrm{Hom}_{\mathcal{A}}(P_\bullet, B) \cong H^*\mathrm{Hom}_{\mathcal{A}}(A, I^\bullet).$$

2. Let $C^{\bullet\bullet}$ be a first quadrant double cochain complex (in an abelian category) such that all rows are acyclic complexes. Prove that the total complex $\mathrm{Tot}(C^{\bullet\bullet})$ is acyclic.

3. Let $H \subseteq G$ be a normal subgroup. Show that $M \mapsto M^H$ defines an additive functor from G-modules to G/H-modules that has an exact left adjoint.

4. Let $C^{\bullet\bullet}$ be a first quadrant double cochain complex of vector spaces over a field k, and let (E_n, d_n) be one of the associated spectral sequences. Suppose that for some r, the vector space $\bigoplus_{p,q} E_r^{p,q}$ is finite dimensional. Prove that the vector space $\bigoplus_n H^n(\mathrm{Tot}(C^{\bullet\bullet}))$ is finite dimensional as well and that

$$\sum_{p,q}(-1)^{p+q}\dim_k E_r^{p,q} = \sum_n (-1)^n \dim_k H^n(\mathrm{Tot}(C^{\bullet\bullet})).$$

5. Let $E_2^{p,q} \implies H^{p+q}$ be a (bounded convergent) first quadrant cohomological spectral sequence. Show that there is an exact sequence

$$0 \to E_2^{1,0} \to H^1 \to E_2^{0,1} \to E_2^{2,0} \to H^2.$$