Math 212 **Homework 2** **Due February 9th, 2009**

1. Compute $\mathrm{Tor}_*^{\mathbb{Z}/4}(\mathbb{Z}/2, \mathbb{Z}/2)$ and $\mathrm{Ext}^*_{\mathbb{Z}/4}(\mathbb{Z}/2, \mathbb{Z}/2)$.

2. Suppose $\mathcal{A}$ is an abelian category such that all objects are projective. Show that in this case all objects are also injective.

3. Suppose that $\mathcal{A}$ is an abelian category with enough injectives such that given any epimorphism $I \to A \to 0$ in $\mathcal{A}$ with I injective, A is also injective (for example, $\mathcal{A}$ could be the category of modules over a PID). Let $C^\bullet$ a bounded cochain complex in $\mathcal{A}$. Show that there is a homomorphism of complexes $C^\bullet \to H^*(C^\bullet)$ that induces an isomorphism on cohomology (here the cohomology is viewed as a complex with zero differentials). **Note:** this quasi-isomorphism is not natural.

4. Let $\mathcal{A}$ be an abelian category with enough injectives. Assume that for all objects A and B, $\mathrm{Ext}^1_{\mathcal{A}}(A, B) = 0$. Show that $\mathcal{A}$ has enough projectives.

5. Let $F : \mathcal{A} \to \mathcal{B}$ be an additive functor between abelian categories. Suppose F has an exact left adjoint $G : \mathcal{B} \to \mathcal{A}$. Show that for any injective $I \in \mathcal{A}$, $F(I)$ is injective in $\mathcal{B}$.

6. Let G be a finite group and $\mathbf{Mod}_G$ the abelian category of (right) $\mathbb{Z}G$-modules. Let $H \subseteq G$ be a normal subgroup. Show that the restriction functor $R^G_H : \mathbf{Mod}_G \to \mathbf{Mod}_H$ has an exact left adjoint. (Try the case $H = \{e\}$ first.)