1. Give a statement and proof of the covariant Yoneda lemma (without using the contravariant version, of course).

2. Let $L : C \to D$ be a left adjoint functor, and $F : I \to C$ some functor. Suppose that the colimit $\text{colim}_I F$ exist in $C$. Show that $L(\text{colim}_I F)$ is a colimit for the functor $L \circ F : I \to D$. Does the same statement hold with ”right adjoint” replacing ”left adjoint”, or ”limit” replacing ”colimit”?

3. Show that all small limits exist in the category of sets, and describe the limit of a functor $F : I \to \text{Sets}$ explicitly.

4. Let $(X, \leq)$ be a partially ordered set, and $X$ the corresponding category. Show that the statement ”all small colimits exist in $X$” is equivalent to the statement ”all subsets of $X$ have a least upper bound”.

5. Show that all small colimits exist in the category of abelian groups, and describe the colimit of a functor $F : I \to \text{Ab}$ explicitly.

6. Let $f : G \to H$ be a group homomorphism. Describe a category $I$ and functor $F : I \to \text{Groups}$ such that the kernel of $f$ is the limit of $F$.

7. Let $F$ be a contravariant functor from sets to sets taking colimits to limits. (Meaning: given a functor $X : I \to \text{Sets}$ that has a colimit, $F(\text{colim}_I X)$ is a limit for the functor $F \circ X : I^{\text{op}} \to \text{Sets}$.) Show that $F$ is a representable functor.

8. Show that the forgetful functor from rings to sets is (co-)representable.