1. Let $C$ be a category, and assume that $x$ and $y$ are final objects (resp., initial objects) for $C$. Prove that there is a unique isomorphism $x \to y$ in $C$.

2. Given sets $X$ and $Y$ let $P$ be the category whose objects are pairs of arrows $(T \to X, T \to Y)$ in $\text{Sets}$ and whose morphisms are the obvious commutative diagrams. Prove that $P$ has a final object, and describe it explicitly. Formulate and prove the dual (that is, with arrows reversed in direction) statement.

3. Let $V$ be the category of vector spaces over a field $F$. Let $\Phi$ be the endofunctor of $V$ sending a vector space to its double dual. Show that there is a natural transformation $\text{id}_V \to \Phi$ that becomes a natural isomorphism if restricted to the full subcategory of finite-dimensional vector spaces; however also show that $\Phi$ cannot be a natural isomorphism.

4. Show that right adjoint functors preserve final objects. Do they preserve initial objects?

5. Suppose that $L : \text{Groups} \to \text{Sets}$ is a left adjoint functor. Prove that $L(\mathbb{Z}) = \emptyset$.

6. Show that the forgetful functor $U$ from abelian groups to commutative monoids has a left adjoint $Q$. What is $Q(\mathbb{N})$?

7. Let $G$ be a small connected groupoid. (A groupoid is a category in which every arrow is an isomorphism, and it is called connected if any two objects are isomorphic.) Show that $G$ is equivalent to a group, that is, a groupoid with one object.

8. Let $U : \text{Groups} \to \text{Sets}$ be the forgetful functor. Compute the group of automorphisms of $U$. 