Math 115A **Homework 3** **Due November 7th, 2008**

1. Let $Id : \mathbb{R}^3 \to \mathbb{R}^3$ be the identity linear transformation. For the ordered bases $\mathcal{A}$, $\mathcal{B}$, compute the matrix representation $[Id]_{\mathcal{A}}^{\mathcal{B}}$. (To be clear, the bases are ordered by reading from left to right.) (2 pts each)

a) $\mathcal{A} = \{2e_2, e_3 - e_1, e_1\}$ and $\mathcal{B} = \{e_1, 2e_2, e_3 - e_1\}$.

b) $\mathcal{A} = \{e_1, e_1 + e_2, e_1 + e_2 + e_3\}$ and $\mathcal{B} = \{2e_1 - 3e_2, e_3 + e_2, 4e_2\}$.

2. Prove: If F is a field, V is an F-vector space of dimension n, and $\mathcal{A}$ is any ordered basis of V, then the matrix representation $[Id]_{\mathcal{A}}^{\mathcal{A}}$ of the identity linear transformation is the $n \times n$-identity matrix I_n. (4 pts)

3. Prove: Let F be a field and let V be an F-vector space of dimension n. Suppose $T : V \to V$ is a linear transformation of rank n. Then there are ordered bases $\mathcal{A}$ and $\mathcal{B}$ of V such that $[T]_{\mathcal{A}}^{\mathcal{B}} = I_n$ is the $n \times n$-identity matrix. (4 pts)

4. Let F be a field and let V and W be F-vector spaces. Suppose $v \in V$. Prove that the map $E_v : \mathcal{L}(V, W) \to W$ defined by $E_v(T) = T(v)$ is a linear transformation. Moreover, show that E_v is onto provided $v \neq 0$. (You may assume that V has finite dimension.) (4 pts)

5. Let F be a field and V an F-vector space. The vector space $\mathcal{L}(V, F)$ is called the *dual vector space* of V and also written V^*. Prove that $\dim_F(V) = \dim_F(V^*)$ if V is finite dimensional. (4 pts)