1. Let $Id : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the identity linear transformation. For the ordered bases $\mathcal{A}$, $\mathcal{B}$, compute the matrix representation $[Id]^{\mathcal{B}}_{\mathcal{A}}$. (To be clear, the bases are ordered by reading from left to right.) (2 pts each)

   a) $\mathcal{A} = \{2e_2, e_3 - e_1, e_1\}$ and $\mathcal{B} = \{e_1, 2e_2, e_3 - e_1\}$.

   b) $\mathcal{A} = \{e_1, e_1 + e_2, e_1 + e_2 + e_3\}$ and $\mathcal{B} = \{2e_1 - 3e_2, e_3 + e_2, 4e_2\}$.

2. Prove: If $F$ is a field, $V$ is an $F$-vector space of dimension $n$, and $\mathcal{A}$ is any ordered basis of $V$, then the matrix representation $[Id]^{\mathcal{B}}_{\mathcal{A}}$ of the identity linear transformation is the $n \times n$-identity matrix $I_n$. (4 pts)

3. Prove: Let $F$ be a field and let $V$ be an $F$-vector space of dimension $n$. Suppose $T : V \rightarrow V$ is a linear transformation of rank $n$. Then there are ordered bases $\mathcal{A}$ and $\mathcal{B}$ of $V$ such that $[T]^{\mathcal{B}}_{\mathcal{A}} = I_n$ is the $n \times n$-identity matrix. (4 pts)

4. Let $F$ be a field and let $V$ and $W$ be $F$-vector spaces. Suppose $v \in V$. Prove that the map $E_v : \mathcal{L}(V, W) \rightarrow W$ defined by $E_v(T) = T(v)$ is a linear transformation. Moreover, show that $E_v$ is onto provided $v \neq 0$. (You may assume that $V$ has finite dimension.) (4 pts)

5. Let $F$ be a field and $V$ an $F$-vector space. The vector space $\mathcal{L}(V, F)$ is called the dual vector space of $V$ and also written $V^*$. Prove that $\dim_F(V) = \dim_F(V^*)$ if $V$ is finite dimensional. (4 pts)