Math 115A **Homework 2** **Due October 24th, 2008**

1. For each of the following maps $T : V \to W$, check if T is an F-linear transformation. If yes, write out a proof. If not, prove that. (2 pts each)

a) $F = \mathbb{R}$, $V = \mathbb{R}^3$, $W = \mathbb{R}^2$ and $T(a_1, a_2, a_3) = (a_1 a_2, a_3 + 2a_1)$.

b) $F = \mathbb{C}$, $V = \mathbb{C}^2$, $W = \mathbb{C}$ and $T(z, w) = \overline{z}$.

c) $F = \mathbb{R}$, $V = \mathbf{C}(\mathbb{R})$ the set of continuous real-valued functions on the real numbers, $W = \mathbb{R}$ and $T(f) = \int_0^1 x^2 f(x) dx$.

2. Let $\mathbb{R}^+$ be the set of positive real numbers. (**Warning:** Positive means just that - zero is not in that set.) Show that there is a structure of an $\mathbb{R}$-vector space on $\mathbb{R}^+$ such that the map $\mathbf{exp} : \mathbb{R} \to \mathbb{R}^+$ defined by $\mathbf{exp}(x) = e^x$ is a linear transformation. That is, exhibit an "addition" and "scalar multiplication" on $\mathbb{R}^+$ satisfying the vector space axioms and show that with these definitions, **exp** is $\mathbb{R}$-linear. (3 pts)

3. Suppose $T_1 : U \to V$ and $T_2 : V \to W$ are F-linear transfomations. (3 pts)

a) Prove: if the Null space of T_1 and the null space of T_2 are both zero, then the Null space of $T_2 \circ T_1$ is zero.

b) Show by example that the converse statement to a) does not hold.

4. Let $T : V \to W$ be an F-linear transformation of finite-dimensional vector spaces with Null space $N(T)$, and let $n = \dim(V)$. Prove that there is a basis $\{v_1, \ldots, v_n\}$ of V and an $r \in \{0, \ldots, n\}$ such that $T(v_i) = 0$ for $1 \leq i \leq r$ and $\{T(v_{r+1}), T(v_{r+2}), \ldots, T(v_n)\} \subseteq W$ is linearly independent. (4 pts)

5. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $T(e_1) = e_2$, $T(e_2) = e_3$ and $T(e_3) = e_1$. Write down the matrix for this transformation with respect to the basis $\{e_1 + e_2, e_2, e_3 - e_1\}$. (4 pts)