Math 115A **Homework 1** **Due October 10th, 2008**

1. For each of the following, check if $W \subseteq V$ is an F-subspace of the F-vector space V. If yes, write out a proof. If not, prove that. (2 pts each)

a) $F = \mathbb{R}$, $V = \mathbb{R}^3$ and $W = \{v = (v_1, v_2, v_3) \in V | 2v_1 - v_2 = 1\}$.

b) $F = \mathbb{Q}$, $V = \mathbb{R}$ and $W = \{x \in \mathbb{R} | x\sqrt{2} \in \mathbb{Q}\}$.

c) $F = \mathbb{R}$, $V = \mathbf{C}(\mathbb{R})$ the set of continuous real-valued functions on the real numbers and $W = \{f \in V | \int_0^1 f(x)dx = 0\}$.

2. For each of the following subsets $S \subseteq V$ of the F-vector space V, check if S is linearly independent. Prove you assertions. (2 pts each)

a) $F = \mathbb{Q}$, $V = \mathbb{R}$ and $S = \{1, \sqrt{2}\}$.

b) $F = \mathbb{R}$, $V = \mathbb{C}$ and $S = \{1, i\}$.

c) $F = \mathbb{C}$, $V = \mathbb{C}$ and $S = \{1, i\}$.

3. Let S be a set and write $\mathbb{R}^S$ for the set of all real-valued functions on S. (4 pts)

a) Explain how $\mathbb{R}^S$ has a natural structure as an $\mathbb{R}$-vector space, giving the addition and scalar multiplication and checking the axioms.

b) For $s \in S$, write $\chi_s : S \to \mathbb{R}$ for the function given by $\chi_s(s) = 1$ and $\chi_s(t) = 0$ for $t \neq s$. Show that the set $\{\chi_s | s \in S\} \subseteq \mathbb{R}^S$ is linearly independent.

c) Find an example of a set S such that the set $\{\chi_s | s \in S\} \subseteq \mathbb{R}^S$ is not a basis of the vector space $\mathbb{R}^S$. Prove your assertion.

4. Let V be an F-vector space, and $S \subseteq T \subseteq V$ be subsets. Prove: if S is linearly dependent, then so is T. (4 pts)