

Review of MATH 115A/4, Fall 2008.

General Remarks: Any material covered in class or contained in sections of the text book covered during the course may appear on the final. In particular, if a definition or theorem does not appear in the list of important definitions or theorems below, it does *not* mean you can ignore it. It simply means, in the case of definitions, that it isn't central to understanding the subject; and in the case of theorems, it most likely means the particular result is an easy consequence of one of the theorems marked important. Likewise, the suggested problems from the book do *not* exhaust the kind of question that could appear on the final exam.

Chapter 1.

(a) *Important Definitions:* Vector space over a field. Subspace. Linear combination. Linear dependence and independence. Basis of a vector space. Dimension of a vector space.

(b) *Important Theorems:* 1.3., 1.6., 1.7., 1.8., 1.9., 1.10., 1.11.

(c) *Suggested practice problems:* **1.2.:** 1,2,7,8,10,11,12,13,16,20. **1.3.:** 1,8,9,11,14,18. **1.4.:** 1,4,12. **1.5.:** 1,4,9,10,18. **1.6.:** 1,2,4,8,22.

(d) *Keep in mind:* Whether some addition and scalar multiplication define a vector space structure, whether some subset is a subspace, whether some set is linearly independent or a basis - these things may depend on the field F . And: do **not** confuse linearly dependent and linearly independent.

Chapter 2.

(a) *Important Definitions:* Linear Transformation. Null space. Range. One-to-one and onto. The matrix representation with respect to two ordered bases. Coordinate vectors with respect to an ordered basis. Isomorphism. The change of coordinate matrix. Similar matrices.

(b) *Important Theorems:* 2.1., 2.3., 2.4., 2.6., 2.9., 2.11., 2.14., 2.18., 2.20., 2.22.

(c) *Suggested practice problems:* **2.1.:** 1,2,5,9,13,14,16,20. **2.2.:** 1,2,4,8. **2.3.:** 1,3,9,12. **2.4.:** 1,2,15,17. **2.5.:** 1, 2,3.

(d) *Keep in mind:* A linear transformation is determined by how it acts on a basis. For a matrix A , the vector Ae_i , where e_i is the i -th standard basis vector, is the i -th column of A .

Chapter 4.

(a) *Important Definitions:* Determinant

(b) *Important Theorems:* The properties listed in Section 4.4.

(c) *Suggested practice problems:* **4.4.:** 1,2,3.

(d) *Keep in mind:* The determinant of a linear transformation $T : V \rightarrow V$ is

obtained as the determinant of the matrix representation $[T]_{\mathcal{B}}$ for an ordered basis \mathcal{B} of V , and the result does not depend on the choice of ordered basis.

Chapter 5.

(a) *Important Definitions:* Eigenvalue. Eigenvector. Eigenspace. Diagonalizability. Characteristic polynomial. Algebraic and geometric multiplicity of an Eigenvalue.

(b) *Important Theorems:* 5.1., 5.2., 5.4., 5.5., 5.6., 5.8., 5.9.

(c) *Suggested practice problems:* **5.1.:** 1,2,3,6,10,12. **5.2.:** 1,2,3,8,9.

(d) *Keep in mind:* To prove a statement about eigenvalues, it is often preferable to use the definition, as opposed to the description of them as zeroes of the characteristic polynomial. Eigenvectors are always non-zero.

Chapter 6.

(a) *Important Definitions:* Inner product. Inner product space. Norm associated with an inner product. Orthogonal set, orthonormal set, orthonormal basis. Adjoint of a linear transformation. Self-adjoint operators, normal operators.

(b) *Important Theorems:* 6.2., 6.4., 6.5., 6.9., 6.14., 6.15., 6.16.

(c) *Suggested practice problems:* **6.1.:** 1,2,5,10,17. **6.2.:** 1,2. **6.3.:** 1,2,3,7,13. **6.4.:** 1,2.

(d) *Keep in mind:* Eigenvectors of a self-adjoint transformation belonging to distinct eigenvalues are orthogonal. It is easy to calculate coordinate vectors with respect to an orthonormal basis.