Math 115A **Homework 9** **Due March 12th, 2010**

1. Let $(V, <\ ,\ >)$ be a finite dimensional inner product space and $T \in \mathcal{L}(V)$ a linear operator. Prove that $N(T^*T) = N(T)$. (5 pts)

2. Let $(V, <\ ,\ >)$ be an inner product space and let T_1 and T_2 be self-adjoint operators on V. Prove that $T_1 \circ T_2$ is self-adjoint if and only if $T_1 \circ T_2 = T_2 \circ T_1$. (5 pts)

3. Let $(V, <\ ,\ >)$ be a finite-dimensional inner product space and let $T \in \mathcal{L}(V)$ be a normal operator. Prove that $N(T) = N(T^*)$ and $R(T) = R(T^*)$. (5 pts)

4. Let $(V, <\ ,\ >)$ be a complex inner product space of dimension n and let $T \in \mathcal{L}(V)$ be a linear operator. Show that there are subspaces $W_1, \ldots, W_{n-1}$ such that W_i has dimension i, $W_i \subseteq W_{i+1}$ and W_i is T-invariant, for $1 \leq i \leq n-1$. (5 pts)