Math 115A **Homework 6** **Due February 19th, 2010**

1. For each of the following F-vector spaces V and linear transformations $T \in \mathcal{L}(V)$, find all eigenvalues of T and their algebraic and geometric multiplicities. (2 pts each)

a) $F = \mathbb{R}$, $V = M(2 \times 2, \mathbb{R})$, and T the linear transformation that sends a matrix $A \in M(2 \times 2, \mathbb{R})$ to its transpose $T(A) = A^t$.

b) $F = \mathbb{C}$, $V = \mathbb{C}^2$, and T the linear transformation given by $T(z, w) = (z + w, z - w)$.

c) $F = \mathbb{R}$, $V = \mathcal{P}_2(\mathbb{R})$, and T given by $T(f)(x) = xf'(x) + f(x)$.

2. Let V be a finite-dimensional F-vector space and $T \in \mathcal{L}(V)$ a linear transformation such that there exists a natural number n with $(T - id)^n = 0$. (Here, for a linear transformation S, S^n means the n-fold composition of S with itself.) Find all the eigenvalues of T. (4 pts)

3. Let V be an F-vector space of dimension n and let $T \in \mathcal{L}(V)$ be a linear transformation with n distinct eigenvalues. (Recall that this implies that T is diagonalizable.) Further, let $S \in \mathcal{L}(V)$ such that $S \circ T = T \circ S$. Show that S is diagonalizable. (5 pts)

4. Let V be a finite-dimensional F-vector space and $T \in \mathcal{L}(V)$ a linear transformation such that $T^2 = T$ and $T \neq 0$ and $T \neq id$. Find all eigenvalues of T. (5 pts)