1. Let $V$ be a finite-dimensional $F$-vector space of dimension $n$ and $T : V \to V$ a linear transformation. Show that $T$ is an isomorphism if and only if there exist ordered bases $\mathcal{A}$ and $\mathcal{B}$ such that $[T]_{\mathcal{B}}^{\mathcal{A}} = I_n$ (here $I_n$ is the $n \times n$-identity matrix). (5 pts)

2. Let $V$ and $W$ be finite-dimensional $F$-vector spaces and suppose that $T : V \to W$ is an isomorphism with inverse $T^{-1}$. Show that the map $\Phi_T : \mathcal{L}(V) \to \mathcal{L}(W)$ given by $\Phi_T(R) = T \circ R \circ T^{-1}$ for $R \in \mathcal{L}(V)$ is an isomorphism. (5 pts)

3. Let $W \subseteq V$ be a subspace of a finite-dimensional $F$-vector space. Show that there is a linear transformation $P_W : V \to V$ such that $P_W \circ P_W = P_W$ and for all $w \in W$, $P_W(w) = w$. (This is called a projector onto $W$.) What is the rank of $P_W$? (5 pts)

4. Let $V$ be a finite-dimensional $F$-vector space. The vector space $V^* = \mathcal{L}(V, F)$ is called the dual vector space of $V$. Show that the map $i : V \to V^{**} = \mathcal{L}(V^*, F)$ to the dual of the dual vector space such that $i(v)(f) = f(v)$ for $v \in V$ and $f : V \to F$ is an isomorphism. (5 pts)