

1. Let V be a finite-dimensional F -vector space of dimension n and $T : V \rightarrow V$ a linear transformation. Show that T is an isomorphism if and only if there exist ordered bases \mathcal{A} and \mathcal{B} such that $[T]_{\mathcal{A}}^{\mathcal{B}} = I_n$ (here I_n is the $n \times n$ -identity matrix). (5 pts)
2. Let V and W be finite-dimensional F -vector spaces and suppose that $T : V \rightarrow W$ is an isomorphism with inverse T^{-1} . Show that the map $\Phi_T : \mathcal{L}(V) \rightarrow \mathcal{L}(W)$ given by $\Phi_T(R) = T \circ R \circ T^{-1}$ for $R \in \mathcal{L}(V)$ is an isomorphism. (5 pts)
3. Let $W \subseteq V$ be a subspace of a finite-dimensional F -vector space. Show that there is a linear transformation $P_W : V \rightarrow V$ such that $P_W \circ P_W = P_W$ and for all $w \in W$, $P_W(w) = w$. (This is called a projector onto W .) What is the rank of P_W ? (5 pts)
4. Let V be a finite-dimensional F -vector space. The vector space $V^* = \mathcal{L}(V, F)$ is called the dual vector space of V . Show that the map $i : V \rightarrow V^{**} = \mathcal{L}(V^*, F)$ to the dual of the dual vector space such that $i(v)(f) = f(v)$ for $v \in V$ and $f : V \rightarrow F$ is an isomorphism. (5 pts)