Math 115A **Homework 3** **Due January 29nd, 2010**

1. For each of the following linear transformations T, compute a basis for the Null space $N(T)$ and determine the dimension of the range $R(T)$. (2 pts each)

a) $F = \mathbb{R}$, $V = W = \mathbb{R}^3$ and $T : V \to W$ given by $T(x, y, z) = (x - y, 0, z)$.

b) $F = \mathbb{R}$, $V = \mathbb{R}^4$, $W = C(\mathbb{R})$ and $T : V \to W$ where $T(v_1, v_2, v_3, v_4)$ is the function $f(x) = v_1 x - v_2 e^x + (v_3 - v_4)\sin(x)$.

c) $F = \mathbb{C}$, $V = \mathbb{C}^2$, $W = \mathbb{C}^3$ and $T : V \to W$ given by $T(z, w) = (2z - w, w - z, w + z)$.

2. Let $V = \mathcal{P}_2(\mathbb{R})$ be the $\mathbb{R}$-vector space of polynomials over $\mathbb{R}$ of degree at most 2, and let $W = \mathbb{R}^3$. Let $\mathcal{A}$ be the ordered basis (ordering from left to right) $\{1, 1+x, 1+x+x^2\}$ of V, and let $\mathcal{B}$ be the standard ordered basis. For each of the following linear transformations T compute its matrix representation with respect to these bases. (2 pts each)

a) $T : V \to W$ given by $T(f) = (f(0), f'(0), f''(0))$.

b) $T : V \to W$ given by $T(f) = (f(1), f(2), f(3))$.

c) $T : W \to V$ given by $T(a, b, c) = a + bx + cx^2$.

3. Let V and W be finite-dimensional F-vector spaces and $T : V \to W$ a linear transformation. Suppose the dimension of the Null space of T is n. Prove that there are ordered bases $\mathcal{A}$ of V and $\mathcal{B}$ of W, respectively, such that the first n columns of the matrix representation $[T]_{\mathcal{A}}^{\mathcal{B}}$ are zero. (4 pts)

4. Let $T : V \to W$ be an F-linear transformation of finite-dimensional vector spaces and assume that $N(T) = 0$. Let $\mathcal{A}$ be an ordered basis of V and $\mathcal{B}$ an ordered basis of W. Prove that the columns of $[T]_{\mathcal{A}}^{\mathcal{B}}$ are linearly independent. (4 pts)