1. For each of the following linear transformations $T$, compute a basis for the Null space $N(T)$ and determine the dimension of the range $R(T)$. (2 pts each)

a) $F = \mathbb{R}$, $V = W = \mathbb{R}^3$ and $T : V \rightarrow W$ given by $T(x, y, z) = (x - y, 0, z)$.

b) $F = \mathbb{R}$, $V = \mathbb{R}^4$, $W = C(\mathbb{R})$ and $T : V \rightarrow W$ where $T(v_1, v_2, v_3, v_4)$ is the function $f(x) = v_1x - v_2e^x + (v_3 - v_4)\sin(x)$.

c) $F = \mathbb{C}$, $V = \mathbb{C}^2$, $W = \mathbb{C}^3$ and $T : V \rightarrow W$ given by $T(z, w) = (2z - w, w - z, w + z)$.

2. Let $V = P_2(\mathbb{R})$ be the $\mathbb{R}$-vector space of polynomials over $\mathbb{R}$ of degree at most 2, and let $W = \mathbb{R}^3$. Let $\mathcal{A}$ be the ordered basis (ordering from left to right) $\{1, 1 + x, 1 + x + x^2\}$ of $V$, and let $\mathcal{B}$ be the standard ordered basis. For each of the following linear transformations $T$ compute its matrix representation with respect to these bases. (2 pts each)

a) $T : V \rightarrow W$ given by $T(f) = (f(0), f'(0), f''(0))$.

b) $T : V \rightarrow W$ given by $T(f) = (f(1), f(2), f(3))$.

c) $T : W \rightarrow V$ given by $T(a, b, c) = a + bx + cx^2$.

3. Let $V$ and $W$ be finite-dimensional $F$-vector spaces and $T : V \rightarrow W$ a linear transformation. Suppose the dimension of the Null space of $T$ is $n$. Prove that there are ordered bases $\mathcal{A}$ of $V$ and $\mathcal{B}$ of $W$, respectively, such that the first $n$ columns of the matrix representation $[T]_{\mathcal{A}}^{\mathcal{B}}$ are zero. (4 pts)

4. Let $T : V \rightarrow W$ be an $F$-linear transformation of finite-dimensional vector spaces and assume that $N(T) = 0$. Let $\mathcal{A}$ be an ordered basis of $V$ and $\mathcal{B}$ an ordered basis of $W$. Prove that the columns of $[T]_{\mathcal{A}}^{\mathcal{B}}$ are linearly independent. (4 pts)