Math 115A **Homework 2** **Due January 22nd, 2010**

1. For each of the following functions T, determine if T is F-linear. Prove your answer. (2 pts each)

a) $F = \mathbb{R}$, $V = W = \mathbb{R}^3$ and $T : V \to W$ given by $T(v) = 2v - (1, 0, 0)$.

b) $F = \mathbb{C}$, $V = \mathbb{C}$, $W = \mathbb{R}$ and $T : V \to W$ given by $T(a + bi) = a - b$.

c) $F = \mathbb{R}$, $V = C^1(\mathbb{R})$, $W = C(\mathbb{R})$ and $T : V \to W$ given by $T(f) = f' - f$.

2. Find a basis for each of the following vector spaces. Prove your answer. (2 pts each)

a) $F = \mathbb{R}$, $V = \{a + bi \in \mathbb{C} | a + b = 0\}$.

b) $F = \mathbb{R}$, $V = \{f \in C^1(\mathbb{R}) | f' = 0\}$.

c) $F = \mathbb{R}$, $V = \mathbb{C}^2$.

3. Let $T_1 : V \to W$ and $T_2 : W \to U$ be F-linear transformations. Suppose the Null space $N(T_2 \circ T_1) = 0$. Prove that $N(T_1) = 0$. Show by example that $N(T_2)$ need not be zero. (3 pts)

4. Let $T : V \to W$ be an F-linear transformation and assume that $N(T) = 0$. Show that there exists an F-linear transformation $S : W \to V$ such that $S \circ T = id_V$. (5 pts)