Math 115A **Homework 1** **Due January 15th, 2010**

1. For each of the following, check if $W \subseteq V$ is an F-subspace of the F-vector space V. If yes, write out a proof. If not, prove that. (2 pts each)

a) $F = \mathbb{R}$, $V = \mathbb{R}^3$ and $W = \{v = (v_1, v_2, v_3) \in V | v_1 + 3v_3 = v_2\}$.

b) $F = \mathbb{Q}$, $V = \mathbb{C}$ and $W = \{x + yi \in \mathbb{C} | x \in \mathbb{Q}\}$.

c) $F = \mathbb{R}$, $V = C^1(\mathbb{R})$ the space of continuously differentiable real-valued functions on the real line, and $W = \{f \in V | f'(1) = 0\}$.

2. For each of the following subsets $S \subseteq V$ of the F-vector space V, check if S is linearly independent. Prove your answer. (2 pts each)

a) $F = \mathbb{R}$, $V = \mathbb{C}$ and $S = \{1 + i, 1 - i\}$.

b) $F = \mathbb{C}$, $V = \mathbb{C}$ and $S = \{1 + i, 1 - i\}$.

c) $F = \mathbb{R}$, $V = \mathbb{R}^3$ and $S = \{(v_1, v_2, v_3) \in V | v_1^2 + v_2 = 5\}$.

3. Let S and T be linearly independent subsets of an F-vector space V. Assume that $S \cap T = \emptyset$. Is $S \cup T$ necessarily linearly independent? If so, give a proof. If not, give a counter-example. (3 pts)

4. Let V be an F-vector space, U and W two F-subspaces, and $S \subseteq U$ and $T \subseteq W$ linearly independent sets. Suppose that $U \cap W = \{0\}$. Is $S \cup T$ necessarily linearly independent? If so, give a proof. If not, give a counter-example. (5 pts)