Midterm Solution Set
269B, Winter 2008

1. (a) Look for $v_m^n = g^n e^{im\theta}$. Insert into (1) to get

$$\frac{g-1}{k} = -a\,\frac{3-4e^{-i\theta}+e^{-2i\theta}}{2h}$$

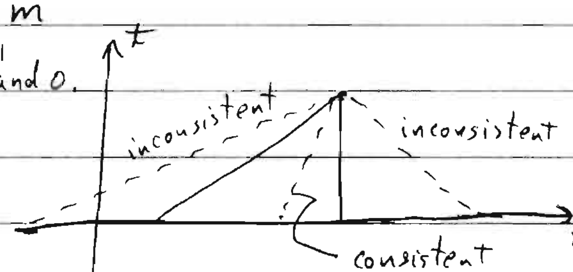
i.e.

$$g = 1 - \frac{a\lambda}{2}\left(3-4e^{-i\theta}+e^{-2i\theta}\right)$$

(b) For the FD scheme v_m^n depends on $v_{m'}^{n'}$ only if

$$m-2(n-n') \le m' \le m$$

The two boundaries have speed $2\lambda^{-1}$ and 0.



Consistency between FD scheme and the PDE requires that the domain of dependence for the PDE is contained in the domain of dependence for the difference scheme, which is only possible if

$$0 < a < 2\lambda^{-1}$$

i.e.

$$a > 0 \quad \text{and} \quad a\lambda > 2$$

must be unstable. Note that there is a gap between the stability requirement from domain of dependence and the more stringent stability requirement from the analysis of g.

2(a) In Lax-Friedrichs

$$\frac{u_m^{n+1} - u_m^n}{k} = u_t + \frac{1}{2} k u_{tt} + O(k^2)$$

$$\frac{u_{m+1}^n + u_{m-1}^n - 2u_m^n}{2k} = \frac{h^2}{2k} u_{xx} + O(h^4/k)$$

$$\frac{u_{m+1}^n - u_{m-1}^n}{2h} = u_x + O(h^2)$$

If $k = O(h)$, then ~~to~~ ~~order~~ keeping terms of size h, we get

$$u_t + \frac{1}{2} k u_{tt} - \frac{h^2}{2k} u_{xx} + a u_x = 0$$

(b) If $k = ch^2$, then the ~~mod~~ scheme is to order h^0 given by

$$u_t \cancel{+ \ldots} - \frac{1}{2c} u_{xx} + a u_x = 0$$

Since this is not the same as $u_t + a u_x = 0$, then the scheme is not consistent.