

Midterm Exam  
Numerical ODEs 269B  
Winter 2008

1. For the one way wave equation  $u_t + au_x = 0$ , consider the finite difference equation

$$\frac{v_m^{n+1} - v_m^n}{k} = -a \frac{3v_m^n - 4v_{m-1}^n + v_{m-2}^n}{2h} \quad (1)$$

- (a) Find the amplification factor  $g(\theta = h\xi)$  for (1).  
(b) Equation (1) is always unstable for  $a < 0$  and has the stability constraint  $a\lambda \leq 1/2$  for  $a > 0$ . Give an explanation for these stability results in terms of the domain of dependence for the PDE and for the finite difference equation.

2. Consider the Lax-Friedrichs scheme

$$\frac{v_m^{n+1} - \frac{1}{2}(v_{m+1}^n + v_{m-1}^n)}{k} = -a \frac{v_{m+1}^n - v_{m-1}^n}{2h} \quad (2)$$

- (a) Find the modified PDE for (2). Assuming that  $k = O(h)$ , you should find the  $O(h)$  modifications to the PDE  $u_t + au_x = 0$ . It would help to first rewrite (2) in the form

$$\frac{v_m^{n+1} - v_m^n}{k} - \frac{(v_{m+1}^n + v_{m-1}^n) - 2v_m^n}{2k} = -a \frac{v_{m+1}^n - v_{m-1}^n}{2h} \quad (3)$$

- (b) Use the modified PDE to argue that the Lax-Friedrichs scheme is not consistent if  $k = O(h^2)$ .