Math 269B: Assignment 2

Assigned Friday Jan 18, due Friday Jan 25

Theory

[1]-[2] Problems 2.2.1, 2.2.2 of Strikwerda.

[3] Show that the leap-frog method is stable for $|a\,\lambda\,| \le 1$ using Von Neumann analysis.

[4] Find the solution of the equation

$$\begin{aligned} u_t + u_x &= 0 \\ u_0(x) &= 0 \quad \mathit{for}\ 2 > |x| > 1 \\ &= 1 - |x| \quad \mathit{for}\ |x| < 1 \end{aligned}$$

with u_0 extended periodically for |x|>2. Note that u is periodic in x, since u_0 is periodic.

Computation

[5] Solve the system in [4]on the interval $0 \le t \le 4$, -2<x<2. For the numerical method use periodic boundary data at x=2 and x=-2. At each time step calculate the L2 error $e^n{}_2$ in the solution at time t=nk, defined by

$$(e_2^n)^2 = \left\| v^n - u^n \right\|_h^2 = h \sum_{m=1}^{N} \left| v_m^n - u_m^n \right|^2$$

in which u is the exact solution from [4]. Plot $e^n{}_2$ as a function of t=nk, and the approximate and exact solutions as functions of x=mh at time 3 for the following values of h= .1 and .01 for λ=0.5 for

(a) the Lax-Friedrichs method
(b) the smoothed forward time/centered space method (2.2.13) in Strikwerda,

Note: You can modify the matlab program from HW1 to do this.

What You Should Turn In

- Answers to the theoretical problems.
- The graphs for the computational problem **5** and a listing of your program.