Final Exam
Numerical ODEs 269A
Fall 2007
Due 5pm Monday Dec 10

1. Consider a one-step method with p-th order accuracy for the ODE

$$\begin{aligned} y'(t) &= f(y,t) \\ y(0) &= y_0. \end{aligned}$$

   Show that the solution has order of accuracy $p+1$, over the first m steps in which m is an integer that is independent of the step size h; i.e.,

$$|y(kh) - y_k| \leq ch^{p+1}$$

   for $0 \leq k \leq m$ and for some constant c, in which $y(t)$ solves the ODE and y_k solves the one-step method. Hint: Recall that p-th order accuracy means that the truncation error d_k is size h^p.

2. Suppose that a Runge-Kutta method of order q is used to supply the initial k values for a p-th order multistep method. What is the minimal value of q that guarantees p-th order accuracy in the resulting solution?

3. Prove that a linear multistep method of the form

$$\sum_{j=0}^{k} \alpha_j y_{n-j} = h \sum_{j=0}^{k} \beta_j f_{n-j}$$

   has the stiff decay property if and only if $\beta_0 \neq 0$ and $\beta_j = 0$ for $j \neq 0$, as stated in Section 5.1.2 of Ascher & Petzold.

4. For the first two BDF methods with coefficients given by Table 5.3, confirm that the boundary of the region of absolute stability intersects the imaginary z axis only at $z = 0$, as shown in Figure 5.6 of Ascher & Petzold.

5. Confirm the formula in Table 5.1 for the truncation error for the 4-th order Adams-Bashforth method.