

Math 269A: Assignment 4

Assigned Monday Oct 29, due Monday November 5

Theory

[1] Consider Newton's method for solution of $f(x)=0$ with

$$f(x) = x^2 - 1$$

Show that the sequence of approximations x_n is monotonically converging if $|x_0|>1$ and monotonically converging after the first step for $|x_0|<1$.

Computation

[2] Implement Newton's method to solve $f(x)=0$ with

$$f(x) = ax^2 + bx + c$$

using the following steps:

- Prescribe n_{\max} , tol , x_0 and set $x_{\text{old}}=x_0$.
- Loop from 1 to n_{\max}
 - Evaluate $\text{fold}=f(x_{\text{old}})$, $\text{dfold}=f'(x_{\text{old}})$ in subroutines
 - Set $dx = -f(x_{\text{old}})/dfold$
 - If $|dx|<\text{tol}$
 - successful, exit loop
 - If $|dx|>\text{tol}$
 - If $n=n_{\max}$, unsuccessful
 - Set $x_{\text{new}} = x_{\text{old}}+dx$ and $x_{\text{old}}=x_{\text{new}}$
 - Continue loop
- If successful, print out solution
- If unsuccessful, print that out

(b) Apply to $a=1$, $b=0$, $c=-1$ with $x_0=2$ and with $x_0=0.5$.

[3] Implement the backward Euler method using Newton's method to solve the ODE

$$\frac{du}{dt} = -u^2$$
$$u(0) = 1$$

over the time interval $0 < t < 2$. Plot the exact solution and the numerical approximation.

What You Should Turn In

- Answers to the theoretical problems.
- The graphs and answers to the computational problems.