Math 269A: Assignment 3

Assigned Monday Oct 22, due Monday Oct 29

Theory

[1] The classic 4th order Runge-Kutta method is

$$Y_1 = y_{n-1}$$
$$Y_2 = y_{n-1} + \frac{h}{2} f_1$$
$$Y_3 = y_{n-1} + \frac{h}{2} f_2$$
$$Y_4 = y_{n-1} + h f_3$$
$$y_n = y_{n-1} + \frac{h}{6}(f_1 + 2 f_2 + 2 f_3 + f_4)$$

$$f_k = f(T_k, Y_k)$$
$$T_1 = t_{n-1}$$
$$T_2 = T_3 = t_{n-1/2} = t_{n-1} + \frac{h}{2}$$
$$T_4 = t_n$$

Rewrite this in the form $y_n = y_{n-1} + h\Psi(t_{n-1}, y_{n-1}, h)$; i.e., find an expression for the function Ψ.

Computation

[2](a) Implement the RK4 method to solve the problem

$$\frac{dy}{dt} = \frac{3t^2 + 2t + 1}{2(y-1)}$$
$$y(0) = -1$$

from HW1. Plot the numerical and exact solutions on the interval [0,2] for a reasonable choice of time steps.

(b) Compare to the exact solution found in HW1 and use the result to estimate the rate of convergence for RK4 on this problem. Specifically, find the error between the exact solution and numerical solution at t = 2.0 using a reasonable set of timesteps dt.

(c) Apply your program to use RK4 to solve the Fitzhugh-Nagumo system

$$\frac{du}{dt} = -u(u-\theta)(u-1) - v + \delta$$
$$\frac{dv}{dt} = \varepsilon(u - \gamma v)$$
$$u(0) = 0.1$$
$$v(0) = 0.1$$
$$\varepsilon = 0.01, \theta = 0.2, \gamma = 2.5, \delta = 0.112$$

over the time interval 0<t<250.

What You Should Turn In

- Answers to the theoretical problems.
- The graphs and answers to the computational problem**s**.