

Midterm Exam
APPLIED DIFFERENTIAL EQUATIONS 266A
Fall 2006

1. Which of the following are locally well-posed (i.e., in a neighborhood of $t = 0$ and the initial data) and why?

- (a) $u_t = \cos(u)$. $u(0) = 5$.
- (b) $u_{tt} = \sqrt{u}$. $u(0) = 1$. $u_t(0) = 0$.
- (c) $u_{tt} = \sqrt{u_t}$. $u(0) = 1$. $u_t(0) = 0$.

2. Find the general solution for the system $u_t = Au$ for each of the following matrices:

(a) $A = A_1$ with

$$A_1 = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \tag{1}$$

(b) $A = A_2$ with

$$A_2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \tag{2}$$

3. Consider the second order ODE

$$u_{tt} + u_t + (u^2 - b^2) = 0 \tag{3}$$

- (a) Find the stationary points and their type for $|b| > 1/8$.
- (b) Find the stationary points and their type for $|b| < 1/8$.
- (c) Draw a picture of the orbits in the phase plane for $|b| > 1/8$, including any connections between the stationary points. Justify the main features of your drawing.

4. Find the energy and plot the phase plane for the following ODE:

$$u_{tt} = u(1 - u^2) \tag{4}$$